MATHEMATICS 8

Two-Dimensional Geometry

Distance Learning

Module 4

Alberta EDUCATION

Mathematics 8

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Mathematics 8
Student Module Booklet
Module 4
Two-Dimensional Geometry
Learning Technologies Branch
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This document is intended for	
Students	✓
Teachers	✓
Administrators	
Parents	
General Public	
Other	

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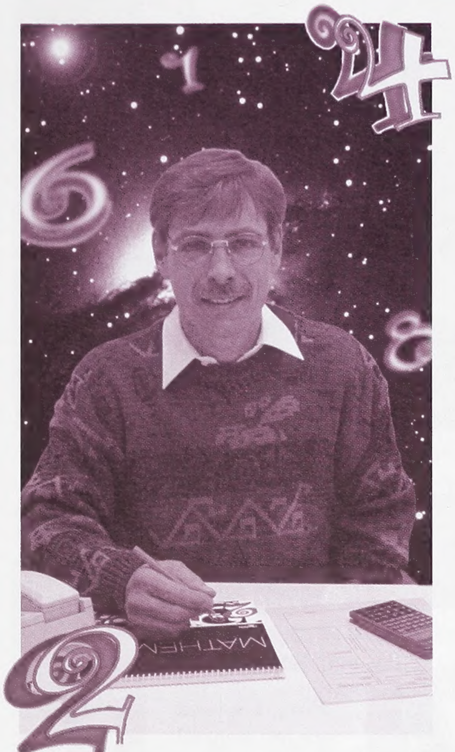
http://ednet.edc.gov.ab.ca/ltb

The use of the Internet is optional. Exploring the electronic information superhighway can be educational and entertaining. However, be aware that these computer networks are not censored. Students may unintentionally or purposely find articles on the Internet that may be offensive or inappropriate. As well, the sources of information are not always cited and the content may not be accurate. Therefore, students may wish to confirm facts with a second source.

Welcome



Welcome to Module 4. We hope you'll enjoy your study of Two-Dimensional Geometry.

Mathematics 8 contains six modules. Work through the modules in the order given, since several concepts build on each other as you progress in the course.

Mathematics 8

Module 1 Number Connections

Module 2 Patterns and Relations

Module 3 Number Applications

Module 4 Two-Dimensional Geometry

Module 5 Statistics and Probability

Module 6 Three-Dimensional Geometry

The document you are presently reading is called a Student Module Booklet. You may find visual cues or icons throughout it. Read the following explanations to discover what each icon prompts you to do.

- Prepare for a problem that will provide a change of topic.

- Prepare for a challenging problem related to the topic of the activity.

- Use the Internet to explore a topic.

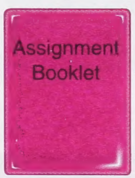
- Use computer software.

- Use a scientific calculator.

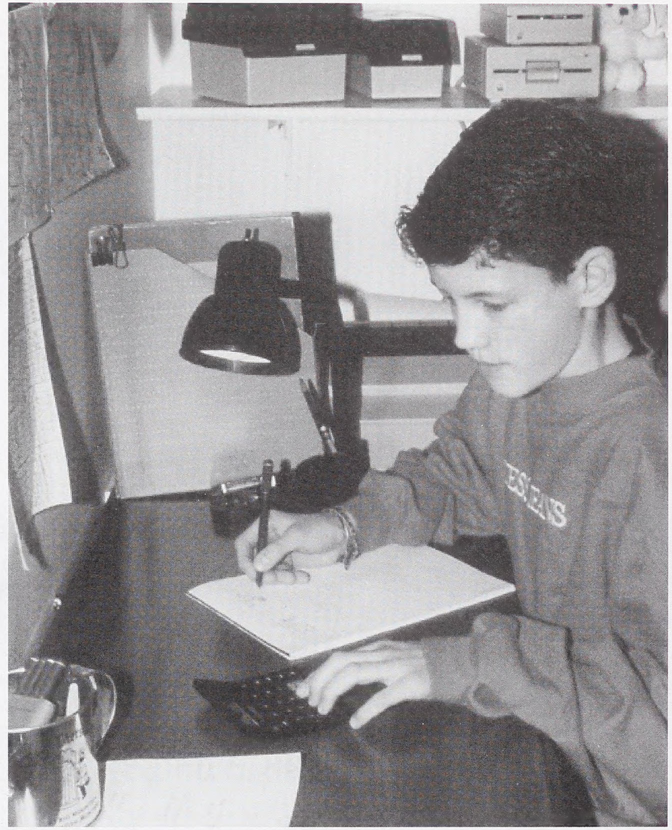
- View a videocassette.

- Pay close attention to important words or ideas.

- Use the suggested answers in the Appendix to correct activities.



- Answer the questions in the Assignment Booklet.



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There are no response spaces provided in this Student Module Booklet. This means that you will need to use your own paper for your responses. You should keep your response pages in a binder so that you can refer to them when you are reviewing or studying.

Technology

Today society is turning to **technology** more than ever before, and it is to your advantage to be able to effectively use technology when required.

Technology is the application of tools, materials, and processes to the solution of problems. More specifically, technology refers to devices and systems that are used in processing, transferring, storing, and communicating information through electronic media.

In Mathematics 8, along with the course materials, you will use a calculator, computer, and videocassette player as tools for learning and doing mathematics.

Calculators are helpful tools for solving problems and exploring patterns and relationships between numbers. Using a calculator will also save you time and help you develop your estimating skills. Therefore, you will be given numerous opportunities in each module to use a calculator.

Computers are useful for organizing and displaying data, or drawing figures. For this reason you will have the chance in many activities to work with popular computer applications such as spreadsheets and draw programs. You will also want to check out the many Internet connections in each module.

Videocassette players allow you to view video programs on key concepts that are difficult to explain in print. That is why video programs are cited in this course.

It is expected that all of you will be able to view the video programs and use a calculator, and that most of you will do the computer activities. However, if you are unable to access a computer, you may do the calculations using a calculator, and draw figures and graphs by hand.

Problem-Solving Skills

One of the exciting features of this course is that you will develop and improve your ability in problem solving. You will need these problem-solving skills many times in your lifetime. Since this course focuses on problem solving, it is important that you understand what a **problem** is.

A problem is a task for which the method of finding the answer (as well as the answer) is not immediately known.

Like any skill, the skill of problem solving must be developed. Problems may or may not involve computation (adding, subtracting, multiplying, and dividing). Some problems are realistic; others are puzzles.

You will have the opportunity in most activities to try a problem-solving challenge. Watch for these icons.

This icon is a cue that the problem will be related to the topic of the activity.

This icon is a cue that the problem will provide a change of topic.

The Four-Stage Process

There are four stages that can be used to solve any problem: understanding the problem, developing a plan, trying the plan, and looking back.

Understanding the Problem

In this stage you should expect to feel puzzled. There are various reasons for feeling this way.

- You may not know the meanings of all the words.
- You may not understand the situation in the problem.
- You may be confused by unnecessary information.

Once you understand the problem, you should think about the problem and make an estimate of what the answer should be. This will help you arrive at a reasonable answer.

Developing a Plan

This is where you should decide on the plan of action that you are going to take to solve the problem. You may consider the following strategies:

- using objects
- using diagrams
- making a table
- working backwards
- using elimination
- using truth tables
- using an equation
- changing your point of view
- making an organized list
- using Venn diagrams
- simplifying a problem
- guessing, checking, and revising
- finding and applying a pattern
- acting out a problem

Note: The Appendix in Module 6 explains these strategies in detail. When you see a problem-solving icon in any module, you should turn to the Module 6 Appendix and review the problem-solving strategies.

Trying the Plan

In this stage you should try the plan and see if it works.

Be sure to work carefully and record your progress. You are encouraged to use a calculator to help with your calculations.

Note: While trying the plan, you should monitor your progress in order to determine if your plan will lead to a solution. You may find that the plan will not produce a solution, in which case a new plan will have to be developed.

Looking Back

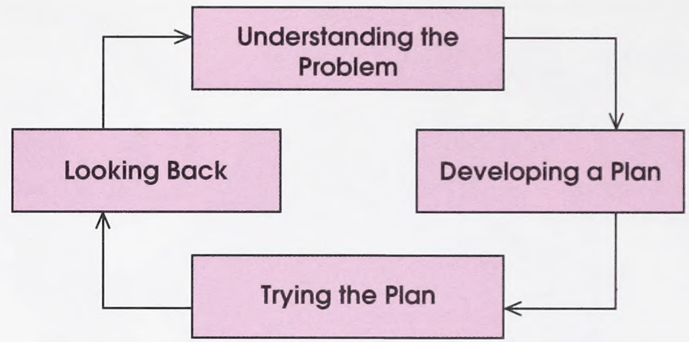
In this stage you should look back at the problem and compare your answer to the estimate you made in the first stage. Restate the problem using your answer.

Ask yourself these questions: "Did my plan work? Is my answer reasonable?"

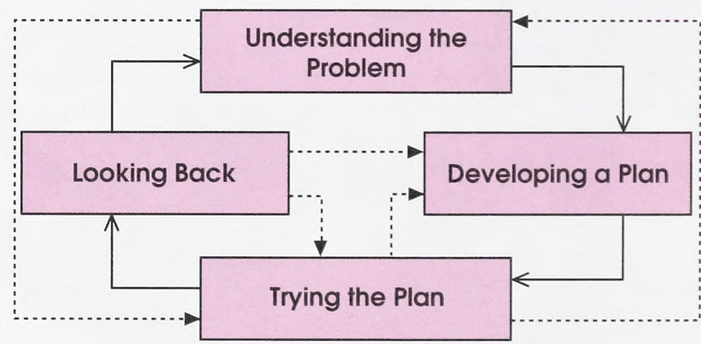
If you did not arrive at an answer, another strategy may work better. If your answer is unreasonable, you may have made errors while trying your plan.

Sequence of Stages

You usually approach a problem in the order outlined in the following diagram.



If you encounter difficulties in your original plan, or if you realize that another strategy will have better results, you may need to return to an earlier stage or use the stages in a different sequence.



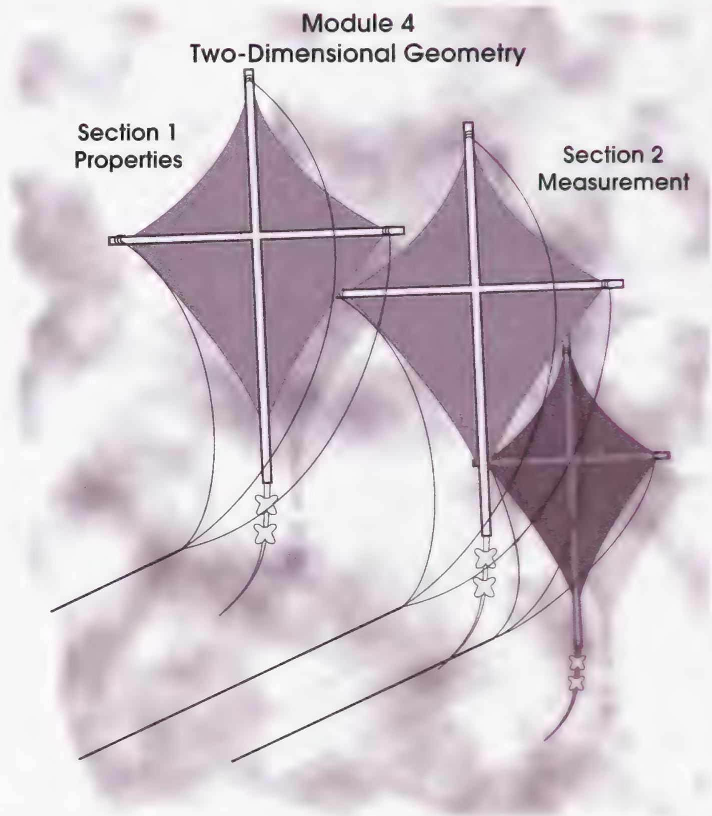
CONTENTS

Module Overview

Most people enjoy flying kites, but kite enthusiasts gain a special pleasure from this activity. Some build their own kites and share kite plans.

Have you ever seen a Sled kite, a Sode kite, a Korean Fighter kite, or a Roller kite? The sails of these kites are of different shapes and sizes. Do you think the area of the sail of a kite affects how it flies?

In this module you will explore two-dimensional geometry. You will discover the properties of various plane (flat) figures and use these properties to solve problems. You will generalize measurement patterns and procedures and solve problems involving perimeter and area.



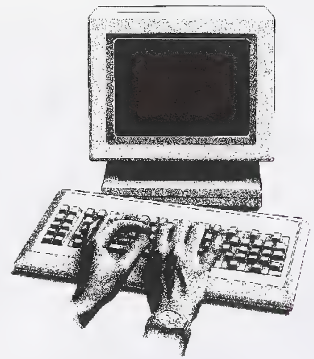
Evaluation

Your mark for this module will be determined by how well you complete the assignments at the end of each section and at the end of the module. In this module you must complete three assignments. The mark distribution is as follows:

Section 1 Assignment	**52 marks**
Section 2 Assignment	**38 marks**
Final Module Assignment	**10 marks**
TOTAL	**100 marks**

When doing the assignments, work slowly and carefully. You must do each assignment independently, but if you are having difficulties, you may review the appropriate section in this module booklet.

If you are working on a computer managed learning (CML) terminal, you will have a module test as well as a module assignment.



Note

There is a final supervised test at the end of this course. Your mark for the course will be determined by how well you do on the module assignments and the supervised final test.

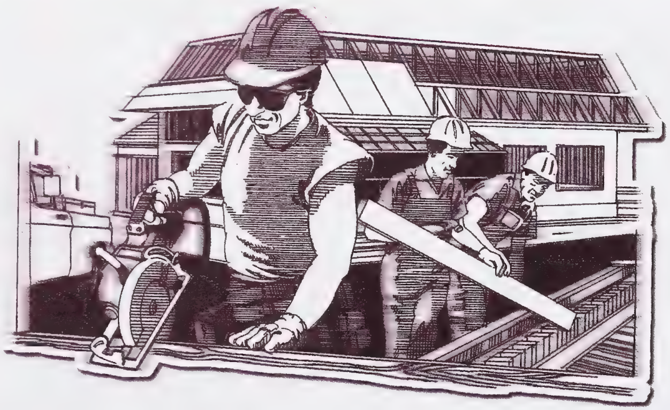
Section 1: Properties

The clock tower in this photograph is part of the Houses of Parliament situated in the heart of London, England. The clock is often mistakenly called Big Ben. The name "Big Ben" actually refers to the huge bell that strikes the hours on the clock.

What two-dimensional figures do you see in this photograph? The clock is in the shape of a circle. Do you see any rectangles, trapezoids, squares, or triangles?

In this section you will investigate the properties of two-dimensional figures—polygons, circles, and special quadrilaterals. You will use these properties to solve problems. You will explore network and colour problems.

Activity 1: Polygons



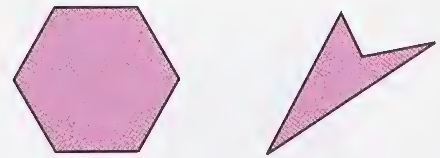
People in most professions use special words. For example, a carpenter may talk about mitre joints, hammers, and beams. A farmer may talk about germination rate, windrows, and augers. An airline representative may talk about departure times, gates, and boarding passes.

Mathematicians also have a specialized vocabulary.

In this activity you will become familiar with terms used to discuss two-dimensional (or flat) shapes. You will also discover the properties of some two-dimensional figures.

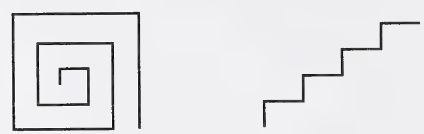
A **polygon** is a simple closed figure with straight sides.

The following figures are polygons. They are simple closed figures with straight sides.

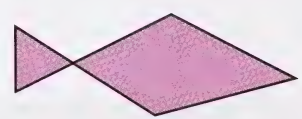


The following figures are **not** polygons; their sides are not straight.

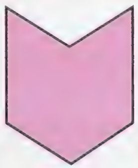
The following figures are **not** polygons; they are not closed. In other words, they do not have an inside and an outside.



The following figure is **not** a polygon; it is not simple. In other words, it has crossovers.



1. Is each of the following figures a polygon? Answer **yes** or **no**. If no, explain why it is not a polygon.

a. 
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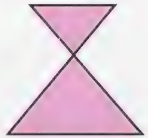
c. 
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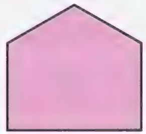
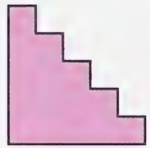
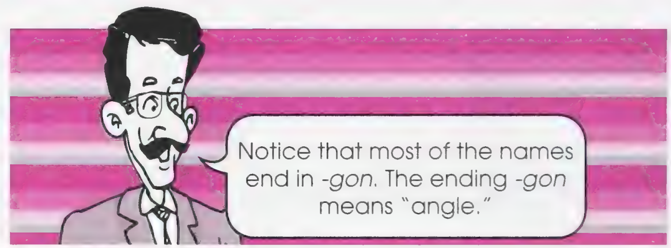
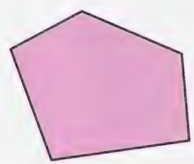
e. 
f. 

Check your answers by turning to the Appendix.

Name of Polygon	Number of Sides	Number of Angles
Triangle	3	3
Quadrilateral	4	4
Pentagon	5	5
Hexagon	6	6
Heptagon	7	7
Octagon	8	8
Nonagon	9	9
Decagon	10	10
Hendecagon	11	11
Dodecagon	12	12

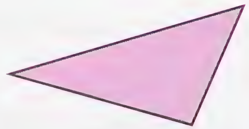


2. State the name that describes each of the following polygons.

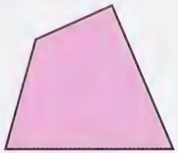
a. 
b. 
c. 

d.

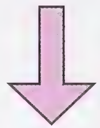
e.



f.

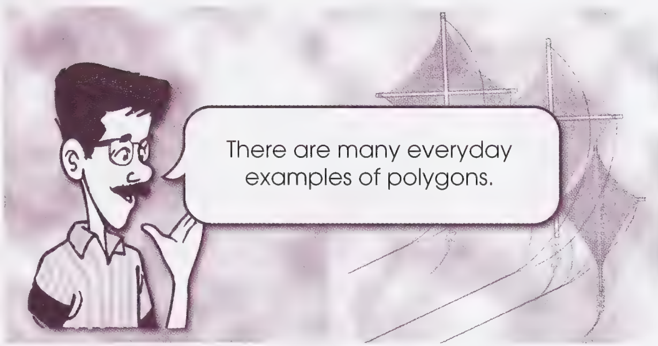


g.



h.

Check your answers by turning to the Appendix.



3. Name the polygon shown in each of the following pictures.

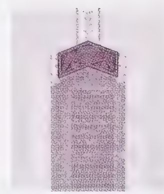
a. a Canadian $1 coin (loonie)

b. a stop sign

c. the end of this barn

d. a playground-zone sign

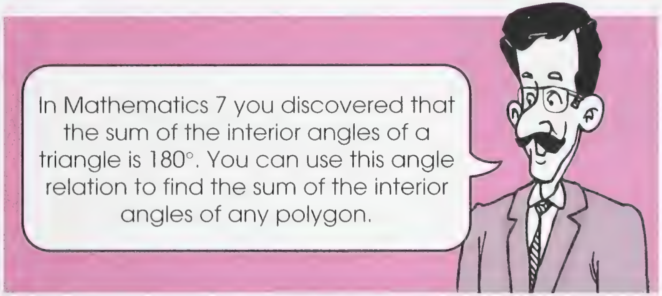
e. the decoration above this door



f. a school-zone sign

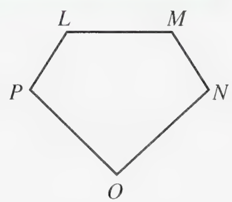
Check your answers by turning to the Appendix.

Investigating Angle Relations in Polygons



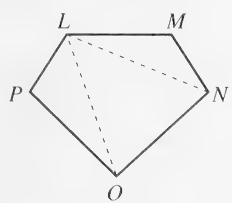
Example 1

Find the sum of the interior angles of figure *LMNOP*.



Solution

Step 1: Draw all the diagonals from one vertex. (A diagonal is a line segment that joins two vertices not already joined by sides.)



Three triangles are formed.

Step 2: Find the sum of the angles of figure *LMNOP*. **Hint:** Multiply the number of triangles by 180°.

$$3 \times 180° = 540°$$

The sum of the angles of figure *LMNOP* is 540°.

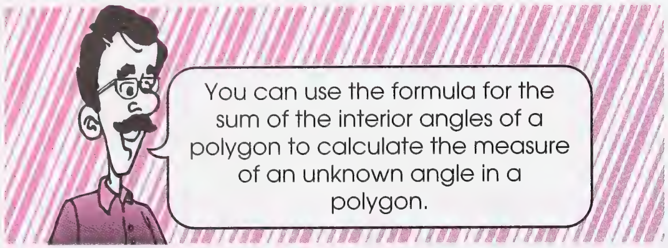
4. a. Copy and complete the following table.

Name of Polygon	Number of Sides	Number of Triangles	Sum of the Interior Angles
Triangle	3	1	180°
Quadrilateral	4		
Pentagon	5	3	540°
Hexagon	6		
Heptagon	7		
Octagon	8		

b. Write a formula to describe how the number of triangles is related to the number of sides. **Note:** Let t represent the number of triangles and n the number of sides.

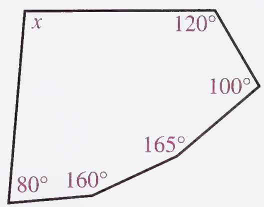
c. Write a formula to describe how the sum of the interior angles of a polygon is related to the number of sides. **Note:** Let s represent the sum of the interior angles (in degrees) and n represent the number of sides.

Check your answers by turning to the Appendix.



Example 2

Find the measure of the unknown angle in this hexagon. Do **not** use a protractor.



Solution

Step 1: The figure is a hexagon; therefore, calculate the sum of the interior angles in a hexagon.

$$\begin{aligned} s &= (n-2)180^\circ \\ &= (6-2)180^\circ \\ &= 4 \times 180^\circ \\ &= 720^\circ \end{aligned}$$

The sum of the angles in a hexagon is 720°.

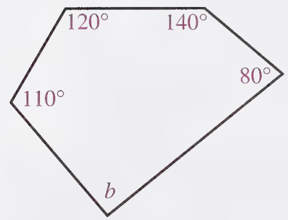
Step 2: Find the measure of the unknown angle.

$$\begin{aligned} x + 120^\circ + 100^\circ + 165^\circ + 160^\circ + 80^\circ &= 720^\circ \\ x + 625^\circ &= 720^\circ \\ x &= 95^\circ \end{aligned}$$

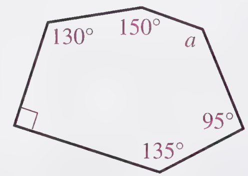
The measure of the unknown angle is 95°.

5. Calculate the measure of the unknown angle in each of the following polygons. Do **not** use a protractor.

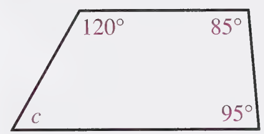
a.



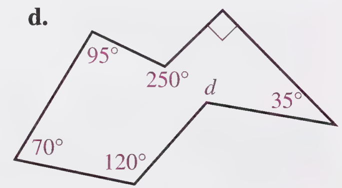
b.



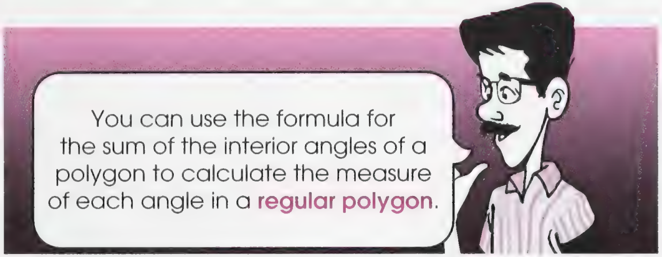
c.

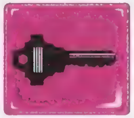


d.



Check your answers by turning to the Appendix.

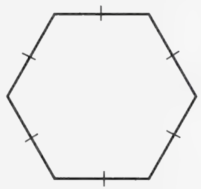




A regular polygon is a polygon with congruent sides and congruent angles. (Congruent means the same size and shape.)

Example 3

What is the measure of each angle in a regular hexagon? Do **not** use a protractor.



Solution

Step 1: Calculate the sum of the interior angles in a hexagon.

$$\begin{aligned} s &= (n-2)180° \\ &= (6-2)180° \\ &= 4(180°) \\ &= 720° \end{aligned}$$

The sum of the interior angles in a hexagon is 720°.

Step 2: Calculate the measure of each interior angle. Because the interior angles are congruent, divide the sum of the angles by 6.

$$720° \div 6 = 120°$$

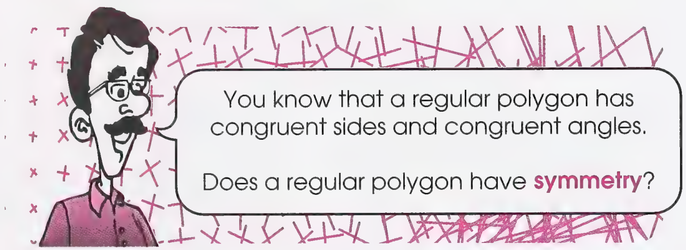
Each interior angle in a regular hexagon is 120°.

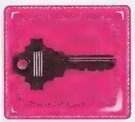
6. Copy and complete the following table.

Name of Polygon	Number of Sides	Sum of the Interior Angles	Measure of Each Interior Angle
Regular Triangle			
Regular Quadrilateral			
Regular Pentagon			
Regular Hexagon			
Regular Heptagon			
Regular Octagon			

Check your answers by turning to the Appendix.

Investigating Symmetry in Regular Polygons





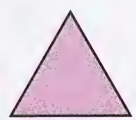
Symmetry is the property that makes a figure look balanced.

You can use tracing paper techniques to test regular polygons for turn symmetry and flip symmetry.

Example

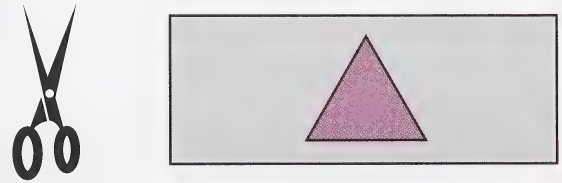
Does a regular triangle have flip symmetry? If so, how many lines of symmetry are there?

Does a regular triangle have turn symmetry? If so, what is the order of turn symmetry?

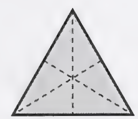


Solution

Step 1: Place tracing paper over the figure and trace the shape. Then cut out the traced figure.

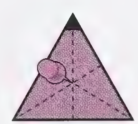


Step 2: Test the figure for flip symmetry. If the tracing can be folded so that one half fits exactly on the other half, the figure has flip symmetry. The fold line is a line of symmetry.

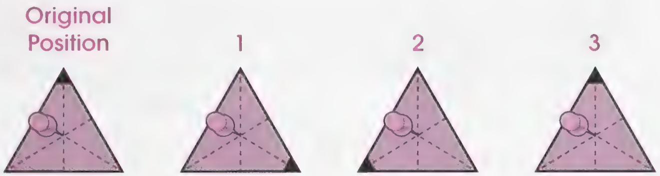


Yes, a regular triangle has flip symmetry; there are three lines of symmetry.

Step 3: Test the figure for turn symmetry. Mark one corner of the traced figure, place the tracing over the original so that the two figures match, put a pin through the centre, and turn the traced figure.



If the traced figure matches the original figure more than once in a full turn, the figure has turn symmetry. The number of times they match in a full turn, not counting the original position, is the order of turn symmetry.



Yes, the figure has turn symmetry; the order of turn symmetry is 3.

7. a. Find the page of regular polygons in the Appendix. Test each polygon for flip symmetry and turn symmetry. Then copy and complete a table like the following.

Name of Polygon	Number of Lines of Symmetry	Order of Turn Symmetry
Regular Triangle	3	3
Regular Quadrilateral		
Regular Pentagon		
Regular Hexagon		
Regular Heptagon		
Regular Octagon		

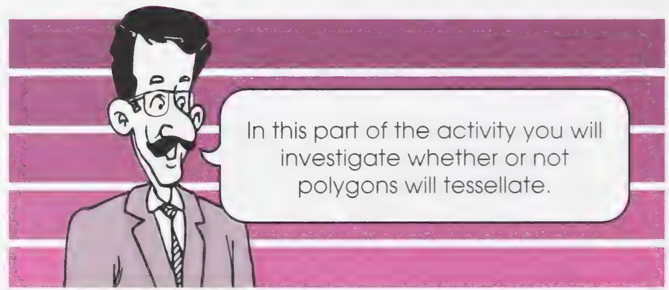
b. What pattern do you notice?

c. From this pattern do you think a regular decagon has flip symmetry? If yes, how many lines of symmetry does it have?

d. Do you think a regular decagon has turn symmetry? If yes, what is the order of turn symmetry?

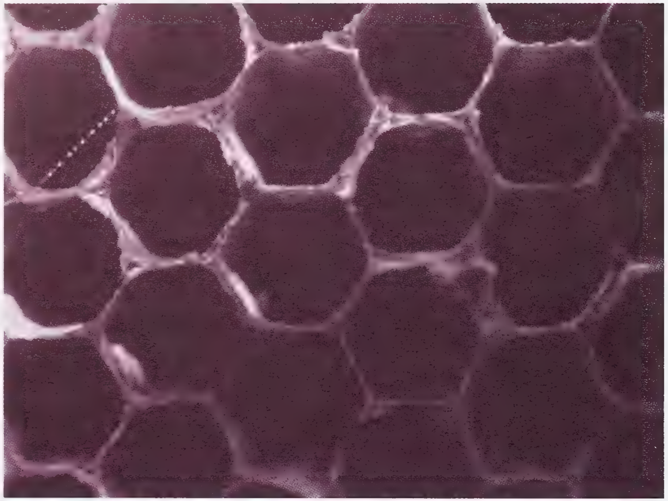
Check your answers by turning to the Appendix.

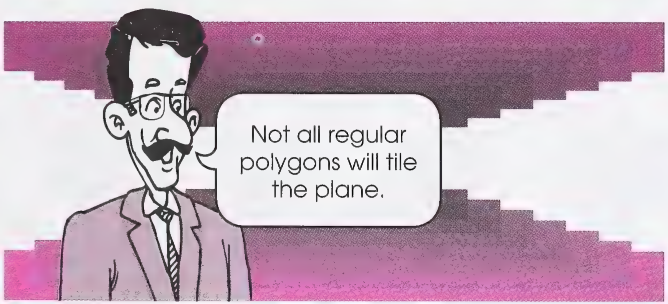
Investigating Whether or Not Polygons Will Tessellate



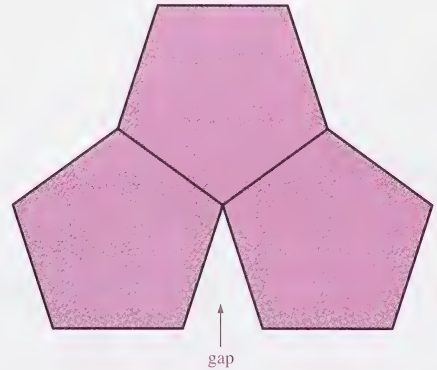
A **tessellation** is a pattern formed by fitting together shapes (without overlapping or leaving spaces) so that they cover the plane. The process of making a tessellation is called **tiling the plane**. Figures that are used to tile the plane are called **tiles**.

A honeycomb is an example of a tessellation in nature. Each of its cells is a regular hexagon.





Congruent regular pentagons do **not** tessellate; there is a gap. If another pentagon were placed in the gap, there would be an overlap.



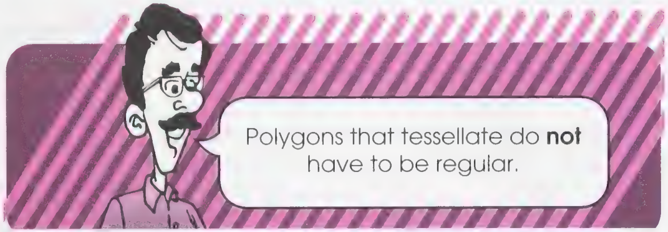
8. **a.** Why do congruent regular hexagons tessellate? **Hint:** Calculate the sum of the angles where the congruent regular hexagons meet.

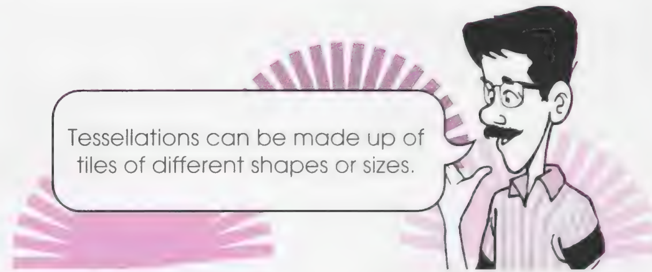
   b. Why don't congruent regular pentagons tessellate? **Hint:** Calculate the sum of the angles where the congruent regular pentagons meet.

9. **a.** Will congruent regular triangles tessellate? Why or why not?

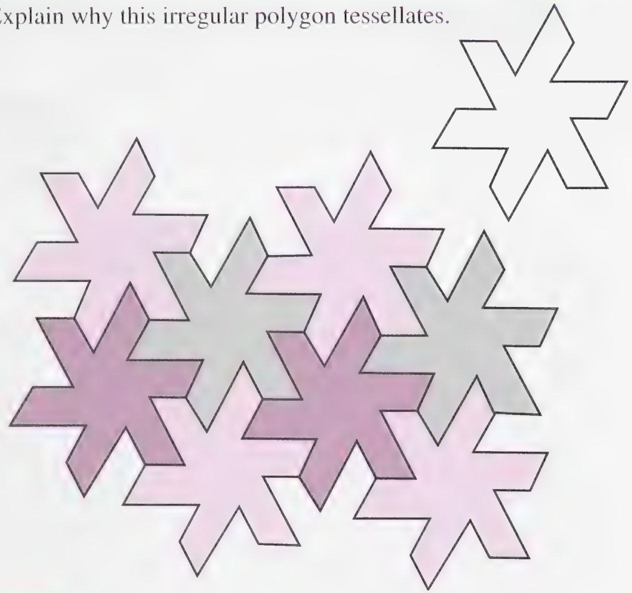
   b. Will congruent regular octagons tessellate? Why or why not?

Check your answers by turning to the Appendix.



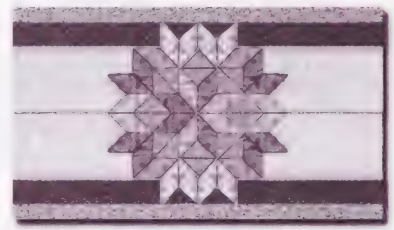


10. Explain why this irregular polygon tessellates.



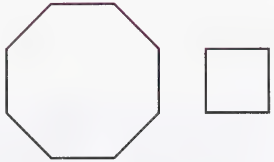
Check your answer by turning to the Appendix.

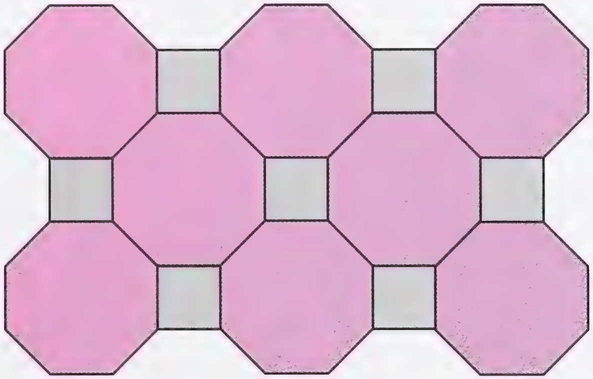
The design in the centre of this quilt is made up of two shapes—congruent quadrilaterals and congruent triangles.



This fountain is covered by different sizes of rectangular tiles.

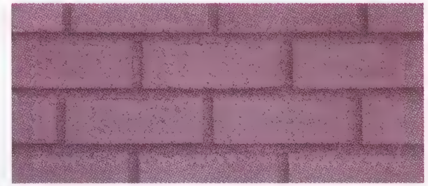
11. Explain why this regular octagon and regular quadrilateral tessellate.



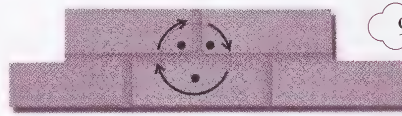


Check your answer by turning to the Appendix.

These rectangular bricks do not meet at their vertices.

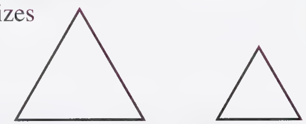


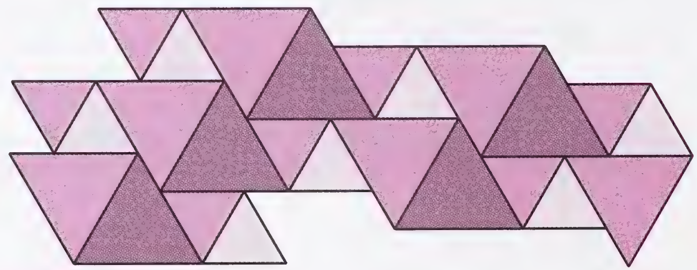
However, the sum of the angles where the bricks meet is 360°.



$$90° + 90° + 180° = 360°$$

12. Explain why these two different sizes of regular triangles tessellate.

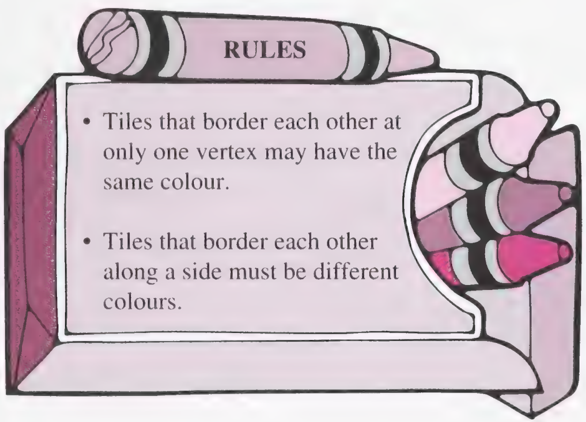




Check your answer by turning to the Appendix.

Colour Problems

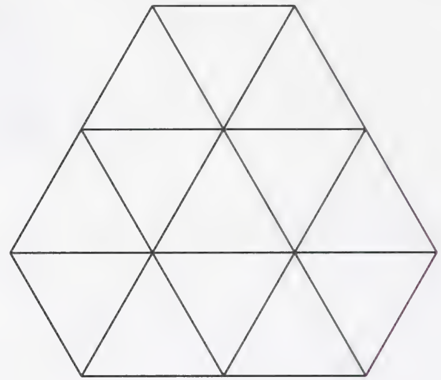
To determine the minimum number of colours required, the following rules are used.



Remember, leaving a tile uncoloured means that you colour it white. A different shade of a colour counts as a different colour.

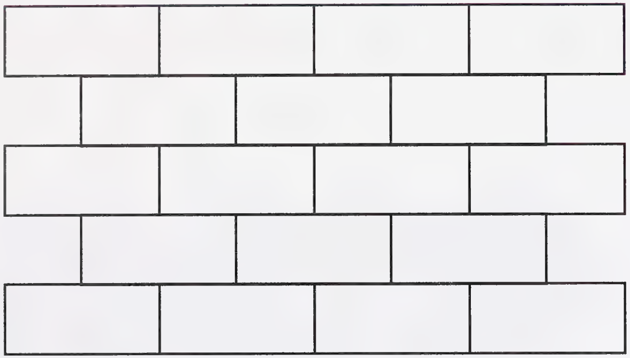
13. For each of the following tessellations, make a tracing and use the given rules to colour the tiles. State the minimum number of colours required for each tessellation. **Hint:** Begin in the middle and work outward.

a.

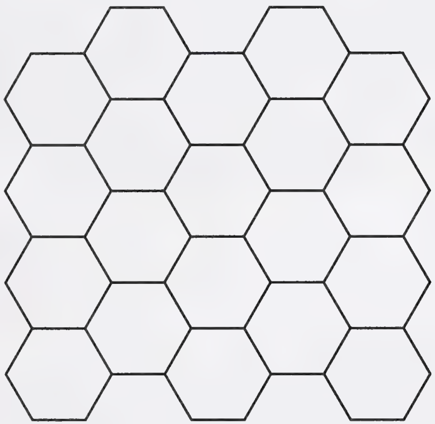


b.

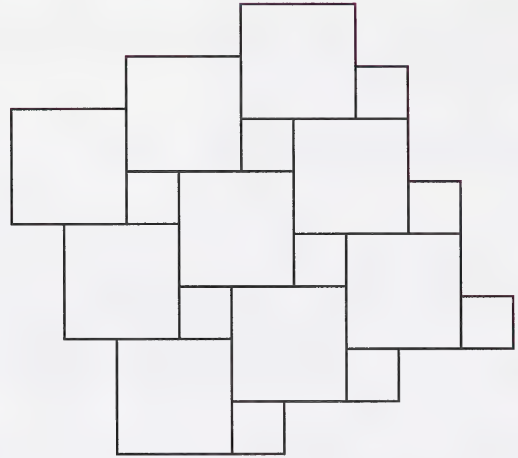
c.



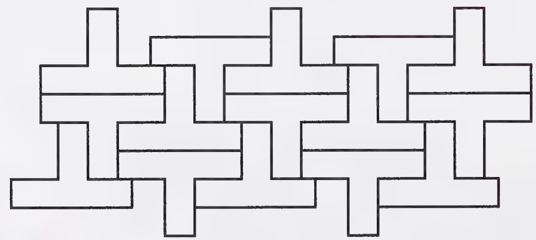
d.



e.



f.



Check your answers by turning to the Appendix.

Did You Know?

Since the time that people began making maps to show different countries, provinces, or states, it has been thought that if you plan well enough, you will need only four colours. For example, the map to the right has only four colours.

Many mathematicians have tried unsuccessfully to prove the four-colour theory. Some worked for years looking for a proof.

In 1976, Kenneth Appel and Wolfgang Hankin, mathematicians at the University of Illinois, announced that they had proved the four-colour theory. Their proof uses over 1000 h of computer time and checks over 10 000 000 cases. It is a proof of contradiction.

Use the Internet to discover more about colouring maps and the four-colour problem. Following is the uniform resource locator (URL) of a site that you may find interesting:

http://www.c3.lanl.gov/mega-math/index.html

Click on The Most Colourful Math of All.

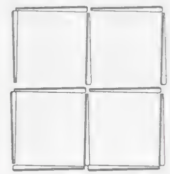
You can download software to work with four-colour problems at the following URL:

http://www.math.ucalgary.ca/~laf/4colors.html

Now Try This

Use a problem-solving strategy to answer the following question.

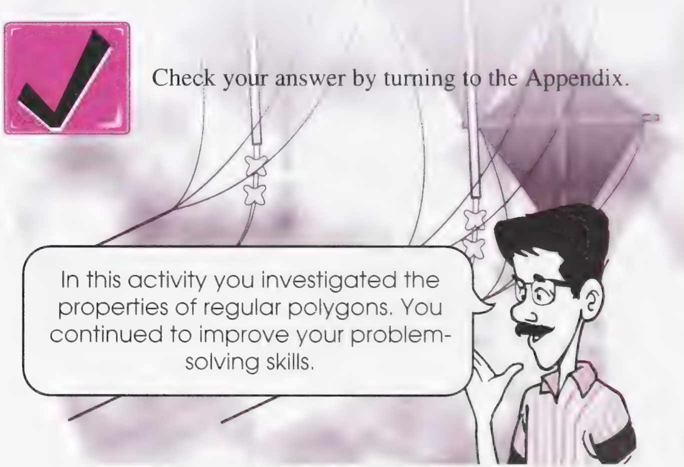
14. Arrange 12 toothpicks in a shape like this.



Notice that there are four small squares and one large square.

Move 2 toothpicks to make a total of seven squares.

Check your answer by turning to the Appendix.

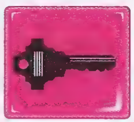


Activity 2: Circles

One of the first things that you see when you wake up in the morning—the face of the dreaded alarm clock—may be in the shape of a circle. There are many other circular shapes in the everyday world. Look around your home for things that are circular.

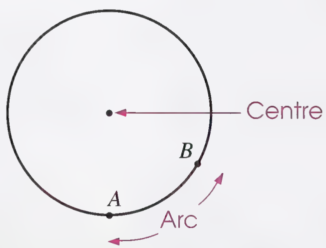
Look around your community for things that are circular.

There are several terms that may be used in discussing circles.

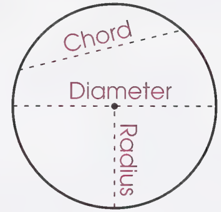


A **circle** is the set of all points in a plane that are the same distance from a fixed point, called the **centre**.

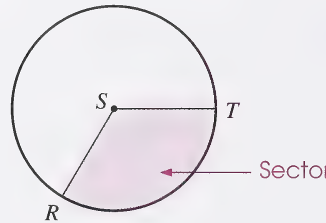
The part of a circle between any two points on the circle is called an **arc**. In the following circle, arc AB is shown.



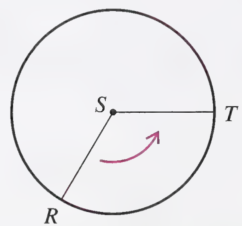
A line segment joining two points on a circle is a **chord**. A chord through the centre of a circle is a **diameter**. The line segment from the centre of a circle to any point on the circle is a **radius**.



A region bounded by a pair of radii and an arc is a **sector**.

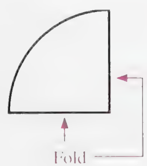


An angle formed by a pair of radii is called a **central angle**. In the following circle, $\angle RST$ is a central angle.

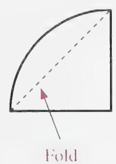


For questions 1 and 2 you will need a compass, protractor, and ruler.

1. Draw a circle with a radius of at least 10 cm. Cut out the circle, and fold the circle into quarters.

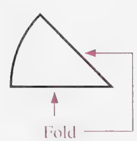


As shown in the following diagram, make a fold.

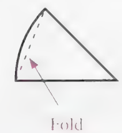


Unfold the circle. What figure is produced by the fold lines? How do you know?

2. Draw a circle with a radius of at least 10 cm. Cut out the circle and fold the circle into eighths.

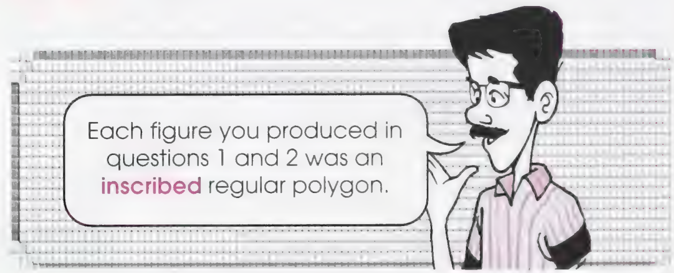


As shown in the following diagram, make a fold.



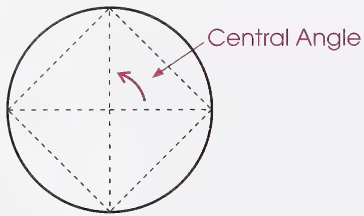
Unfold the circle. What figure is produced by the fold lines? How do you know?

Check your answers by turning to the Appendix.



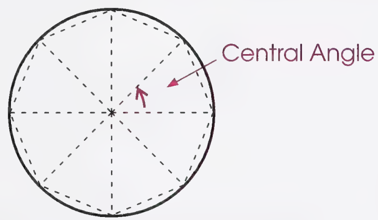
An inscribed figure is one drawn inside another figure so that the two figures have common points but do not intersect. When a polygon is inscribed in a circle, the sides of the polygon are chords of the circle.

Look at the regular quadrilateral (square) that you made in question 1. Notice that there are four congruent central angles.



Each central angle has a measure of 90°. $360° \div 4 = 90°$

Look at the regular octagon that you made in question 2. Notice that there are eight congruent central angles.



Each central angle has a measure of 45°. $360° \div 8 = 45°$

3. If the following number of congruent central angles are made in a circle, what is the measure of each of the central angles?

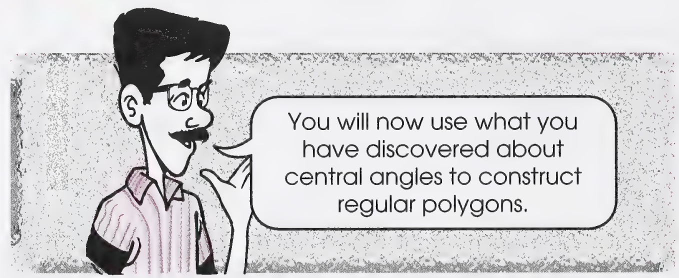
   a. 5 **b.** 6 **c.** 9 **d.** 10

4. If the following number of congruent central angles are drawn in a circle and then the vertices are joined with chords, what kind of regular polygon is created?

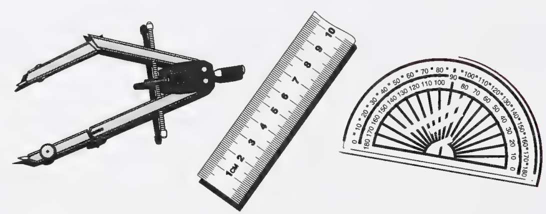
   a. 11 **b.** 5 **c.** 7 **d.** 10

5. What relationship is there between the number of sides in the inscribed regular polygon and the number of central angles drawn?

Check your answers by turning to the Appendix.



You will need a compass, protractor, and straightedge.



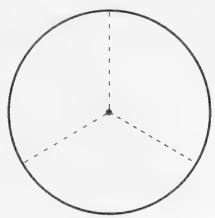
Example 1

Draw a regular triangle using a compass, protractor, and straightedge.

Solution

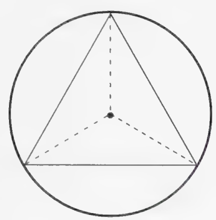
Step 1: Use this reasoning. A regular triangle has three sides, so you will need to draw three congruent central angles.

Step 2: With a compass draw a circle. With a protractor make three congruent angles at the center of the circle. **Hint:** The radius of the circle should be at least 10 cm.



$360° \div 3 = 120°$

Step 3: With a straightedge draw a regular triangle, as shown.



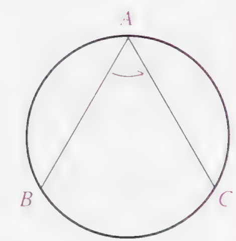
6. Use a compass, protractor, and straightedge to draw each of the following figures.

   a. regular hexagon   **b.** regular pentagon

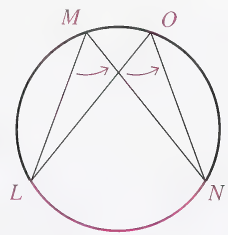
Check your answers by turning to the Appendix.

An inscribed angle is drawn inside a circle so that there are three common points. The arms of an inscribed angle are chords of the circle.

For example, $\angle ABC$ is an inscribed angle. **Note:** Because it is stretched over the arc BC, $\angle ABC$ is said to be "subtended by the arc BC."



In the following diagram, $\angle LMN$ and $\angle LON$ are inscribed angles. Each is stretched over the arc LN.



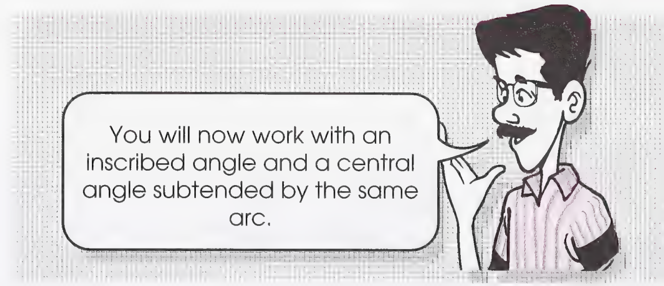
When two or more inscribed angles are stretched over the same arc, they are said to be "subtended by the same arc."

7. With a compass draw two circles. In each circle draw two or more inscribed angles subtended by the same arc. Be sure the arcs are different sizes in each of the circles.

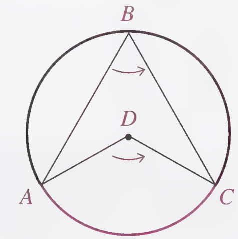
With a protractor measure the angles.

What relationship do you notice between the inscribed angles subtended by the same arc?

Check your answer by turning to the Appendix.



In the diagram to the right, $\angle ABC$ is an inscribed angle and $\angle ADC$ is a central angle. $\angle ABC$ and $\angle ADC$ are each subtended by the arc AC.



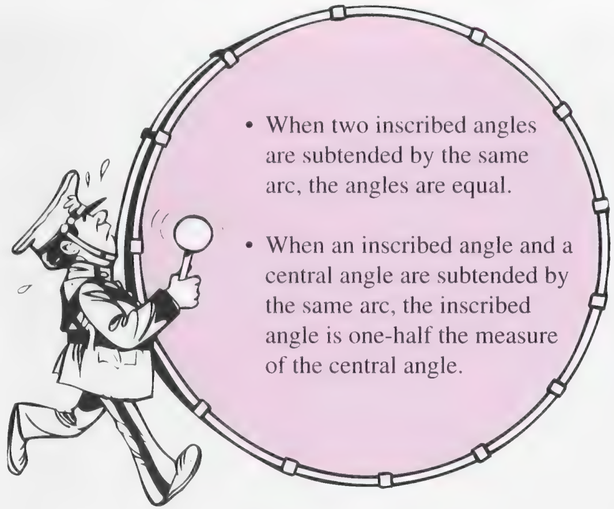
8. With a compass draw three circles. In each circle draw an inscribed angle and a central angle subtended by the same arc. Be sure the arcs are different sizes in each drawing.

With a protractor measure the angles in each drawing.

What relationship do you notice between an inscribed angle and a central angle subtended on the same arc?

Check your answer by turning to the Appendix.

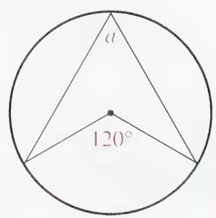
In questions 7 and 8, you discovered two angle relations.



You can use these relations to calculate an unknown angle.

Example 2

Calculate the unknown angle in the given figure. Do **not** use a protractor.



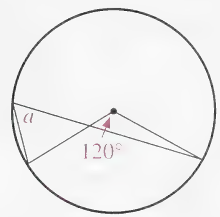
Solution

Statement	Reason
$a = \frac{1}{2} \times 120°$ $\therefore \; a = 60°$	An inscribed angle is one-half the measure of a central angle subtended on the same arc.

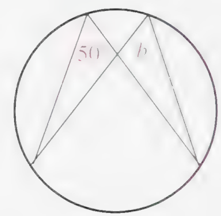
The unknown angle is 60°.

9. Calculate the unknown angle in each of the following figures. Do **not** use a protractor.

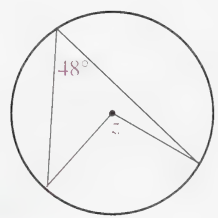
a.



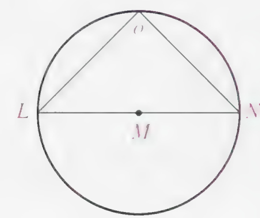
b.



c.



d.



Check your answers by turning to the Appendix.

Did You Know?

Have you ever played the game Trivial Pursuit®? Did you know the game was invented by two Canadians?

Find and read the article entitled "The Game of Trivial Pursuit®" in the Appendix.

10. **a.** Who invented the game of Trivial Pursuit®?

b. When was the game invented?

c. Answer this question; it is a typical trivia question.

How many grooves can be found on one side of a record?

Check your answers by turning to the Appendix.

You may play a modified version of Trivial Pursuit® on the Internet at this uniform resource locator:

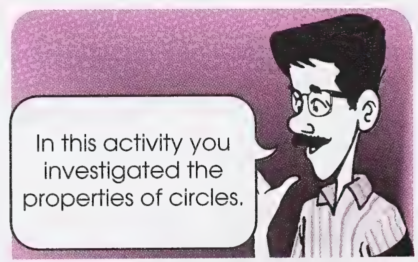
http://www.trivialpursuit.com/htdocs/rules.html

Now Try This

Use a problem-solving strategy to answer the following question.

11. A comedian was so boring that one-half of the audience left after 5 min. A few minutes later, one-third of the remaining audience left. Gradually, one-fourth of those remaining left, leaving only 9 people in the audience. How many people were in the audience at the beginning of the comedian's act?

Check your answer by turning to the Appendix.



Activity 3: Quadrilaterals

Did you know that an animal with four feet is called a quadruped? *Quadr(u)-* means "four" and *ped-* means "foot".

What other words do you know that have the stem *quadr-*? Did you think of quadruplet (one of four offspring), quadriplegic (a person with four limbs paralysed), and quadriceps (the four-part muscle at the front of the thigh)?

In this activity you will investigate the properties of **quadrilaterals**.

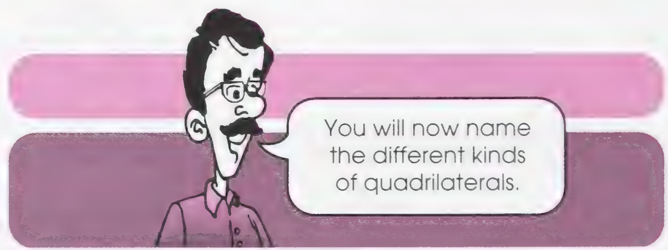
Quadrilaterals are two-dimensional figures having four straight sides and four angles.

1. Find the page of dot paper in the Appendix and photocopy it.

   By joining any of the 9 dots in a 3×3 array of dots, draw 15 **different** quadrilaterals. **Note:** No two shapes should be congruent—the same size **and** shape. For example, the following quadrilaterals are congruent; they are just in different positions. They do **not** count as 4 different figures.

Check your answer by turning to the Appendix.

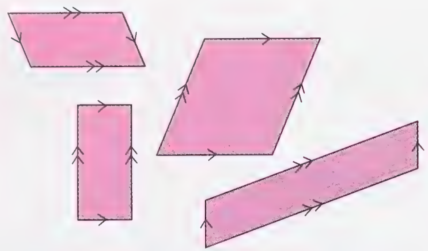
Names of Special Quadrilaterals



Some quadrilaterals have two pairs of parallel sides.

A quadrilateral with two pairs of parallel sides is a **parallelogram**.

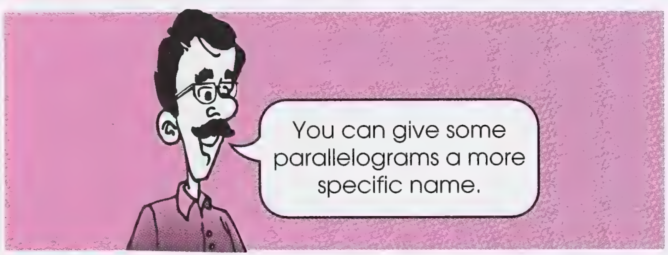
Each of these figures is a parallelogram.



The symbols → and →→ indicate pairs of parallel sides.

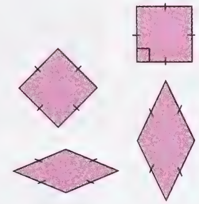
2. Which of the 15 different quadrilaterals in question 1 are parallelograms?

Check your answer by turning to the Appendix.



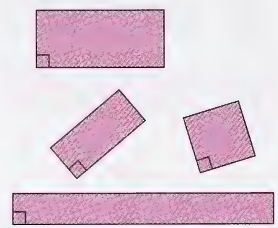
A parallelogram with four congruent sides is a **rhombus**. A parallelogram with a right angle is a **rectangle**. A parallelogram with four congruent sides and a right angle is a **square**.

Each of these parallelograms is a rhombus.



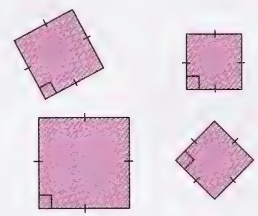
The slashes indicate congruent sides.

Each of these parallelograms is a rectangle.



The symbol ⊾ indicates a right angle.

Each of these parallelograms is a square.



3. **a.** Which of the 15 different quadrilaterals in question 1 are rectangles?

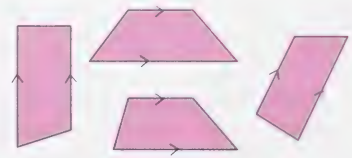
b. Which of the 15 different quadrilaterals in question 1 are rhombuses?

c. Which of the 15 different quadrilaterals in question 1 are squares?

Check your answers by turning to the Appendix.

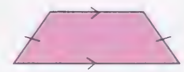
A quadrilateral with exactly one pair of parallel sides is a **trapezoid**.

Each of these figures is a trapezoid.



A trapezoid with the non-parallel sides congruent is called an **isosceles trapezoid**.

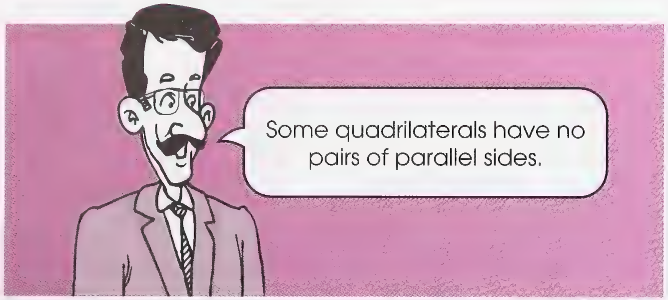
The following figure is an isosceles trapezoid.



4. **a.** Which of the 15 different quadrilaterals in question 1 are trapezoids?

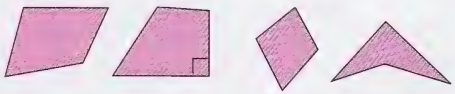
b. Which of the 15 different quadrilaterals in question 1 are isosceles trapezoids?

Check your answers by turning to the Appendix.



A quadrilateral without any parallel sides is called a **trapezium**.

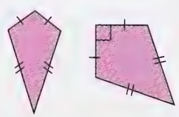
Each of these figures is a trapezium.



Some trapeziums have **adjacent** sides congruent. (Adjacent sides are sides that share a common vertex.)

A **kite** is a **convex** trapezium with two sets of adjacent sides congruent. (A convex figure is a figure in which all the sides turn outward.)

Each of these figures is a kite.

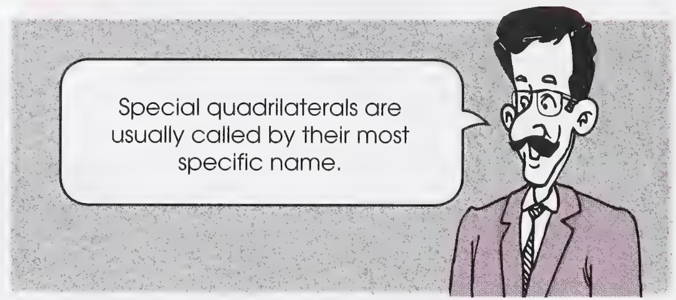


A **dart** is a **concave** trapezium with two sets of adjacent sides congruent. (A concave figure is a figure in which some of the sides turn inward.) A dart is sometimes called an **arrowhead** or a **deltoid**.

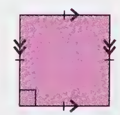
Each of these figures is a dart.

5. **a.** Which of the 15 quadrilaterals in question 1 are trapeziums?
b. Which of the 15 quadrilaterals in question 1 are kites?
c. Which of the 15 quadrilaterals in question 1 are darts?

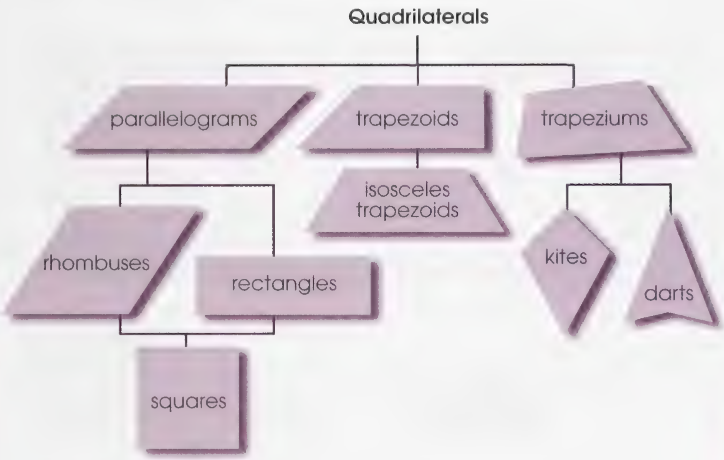
Check your answers by turning to the Appendix.



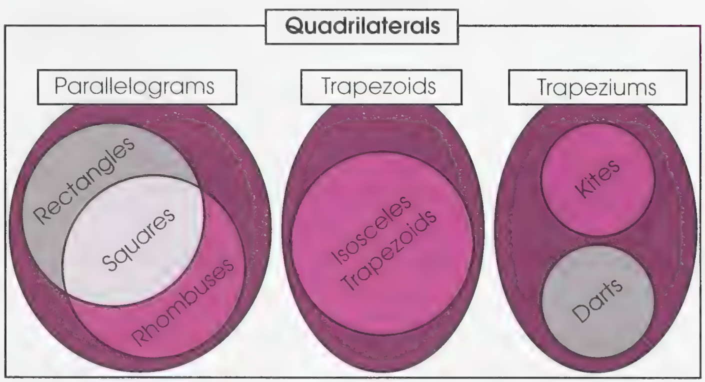
For example, the quadrilateral at the right is usually called a square, rather than a parallelogram or a rhombus.



The relationship between the different kinds of quadrilaterals is shown in the following organizational chart.



The relationship can also be shown with a Venn diagram.

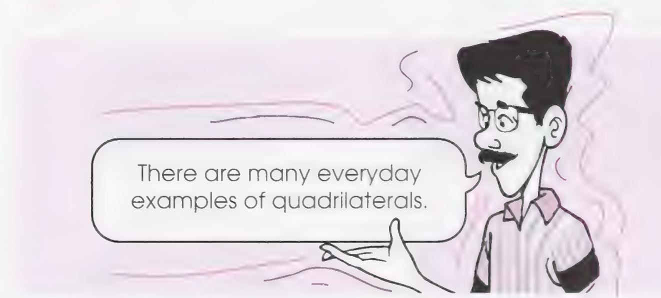


Use the organizational chart and the Venn diagram to answer question 6.

6. Answer **yes** or **no** to these questions.

- **a.** Is every rhombus a parallelogram?
- **b.** Is every square a rectangle?
- **c.** Is every parallelogram a rectangle?
- **d.** Is every rectangle a parallelogram?
- **e.** Is every square a rhombus?
- **f.** Is every rhombus a square?
- **g.** Is every rectangle a square?
- **h.** Is every trapezoid an isosceles trapezoid?
- **i.** Is every trapezium a kite?
- **j.** Is every dart a trapezium?

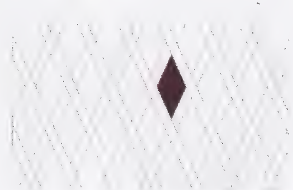
Check your answers by turning to the Appendix.



7. Name the quadrilateral highlighted in each of the following pictures. **Hint:** Remember to use the most specific name.

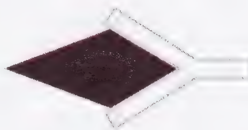
a. the roof of this house

b. a section in a toddler's gate



c. a door panel

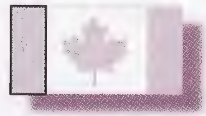
d. a slingshot with the elastic part pulled back



e. the railing section of this staircase

8. Name the outlined quadrilateral in each of the following flags. **Hint:** Remember to use the most specific name.

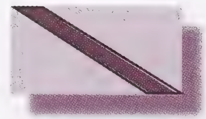
a.



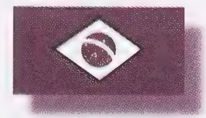
Canada

Kuwait

c.

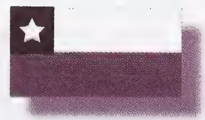


Trinidad and Tobago

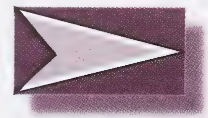


Brazil

e.



Chilie



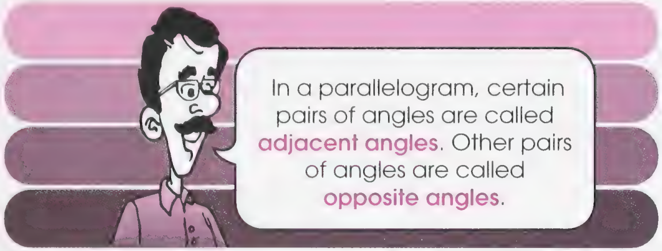
Guyana

Check your answers by turning to the Appendix.

Use the Internet to research other flags of the world. You may find this uniform resource locator (URL) useful.

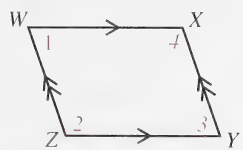
http://immigration-usa.com/flags.html

Investigating Angle Relations in Parallelograms



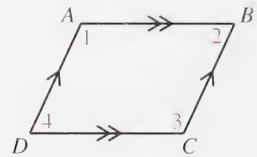
Adjacent angles in a parallelogram share a common side. Opposite angles in a parallelogram are non-adjacent angles.

For example, in parallelogram *WXYZ*, $\angle 1$ and $\angle 2$ are adjacent angles. Other pairs of adjacent angles are $\angle 2$ and $\angle 3$, $\angle 3$ and $\angle 4$, and $\angle 1$ and $\angle 4$.



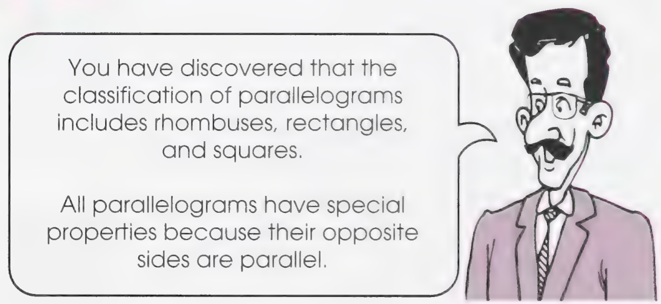
In parallelogram *WXYZ*, $\angle 1$ and $\angle 3$ are opposite angles; $\angle 2$ and $\angle 4$ are also opposite angles.

9. Use this diagram to answer the following questions.



a. Name the opposite angles in parallelogram *ABCD*.
b. Name the adjacent angles in parallelogram *ABCD*.

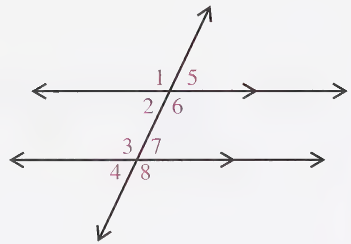
Check your answers by turning to the Appendix.



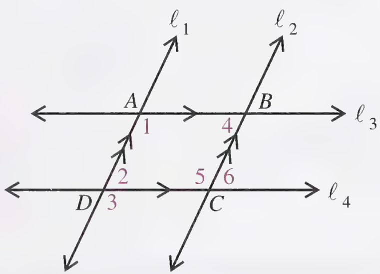
From your work with angles in Mathematics 7, you know the relationships between angles formed by a pair of parallel lines and a transversal. You will use this knowledge to discover relationships between angles in a parallelogram.

You already know that certain pairs of angles formed by a pair of parallel lines and a transversal are equal.

For example, in the figure at the right, $\angle 1$ and $\angle 3$ are equal because they are **corresponding** angles of parallel lines. As well, $\angle 3$ and $\angle 6$ are equal because they are **interior alternate** angles of parallel lines.



10. Parallelogram *ABCD* in the given diagram has been formed by two sets of parallel lines.



a. Explain why $\angle 1 = \angle 3$.

b. Explain why $\angle 3 = \angle 5$.

c. Explain why you can conclude that in parallelogram *ABCD* the opposite angles $\angle 1$ and $\angle 5$ are equal.

d. Explain why $\angle 2 = \angle 6$.

e. Explain why $\angle 6 = \angle 4$.

f. Explain why you can conclude that in parallelogram *ABCD* the opposite angles $\angle 2$ and $\angle 4$ are equal.

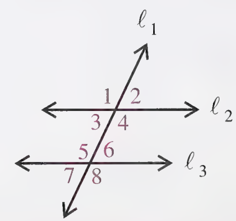
g. What can you conclude about opposite angles in a parallelogram?

Check your answers by turning to the Appendix.

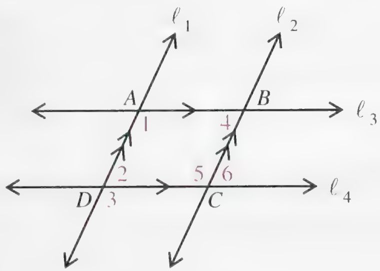
You already know that when a pair of parallel lines is cut by a transversal, some of the angles formed are **supplementary**.

Two supplementary angles have a sum of 180°.

For example, in the following figure, $\angle 1$ and $\angle 2$ are supplementary because they form a straight angle. Other pairs of supplementary angles in the figure are $\angle 1$ and $\angle 3$, $\angle 2$ and $\angle 4$, $\angle 3$ and $\angle 4$, $\angle 5$ and $\angle 6$, $\angle 6$ and $\angle 8$, $\angle 5$ and $\angle 7$, and $\angle 7$ and $\angle 8$.



11. Use this diagram to answer the following questions. **Note:** Parallelogram $ABCD$ in the diagram has been formed by two sets of parallel lines.



a. Explain why $\angle 2$ and $\angle 3$ are supplementary.

b. Explain why $\angle 1 = \angle 3$.

c. Explain why $\angle 3 = \angle 5$.

d. Explain why you can conclude that in parallelogram $ABCD$ the adjacent angles $\angle 1$ and $\angle 2$ are supplementary.

e. Explain why you can conclude that in parallelogram $ABCD$ the adjacent angles $\angle 2$ and $\angle 5$ are supplementary.

f. What can you conclude about adjacent angles in parallelograms?

Check your answers by turning to the Appendix.

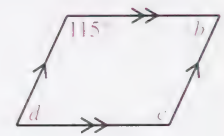
In questions 10 and 11 you discovered two relations:

- Opposite angles of a parallelogram are equal.
- Adjacent angles of a parallelogram are supplementary.

You can use these angle relations to find unknown angles in parallelograms.

Example

Find the measure of each of the unknown angles in the given parallelogram. Do **not** use a protractor.

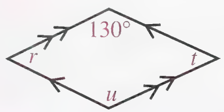


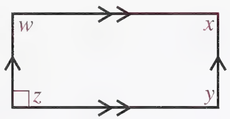
Solution

Statement	Reason
$c = 115°$	Opposite angles of a parallelogram are equal.
$115° + b = 180°$ $\therefore\ b = 65°$	Adjacent angles of a parallelogram are supplementary.
$d = 65°$	Opposite angles of a parallelogram are equal.

$\therefore\ b = 65°,\ c = 115°,\ \text{and}\ d = 65°$

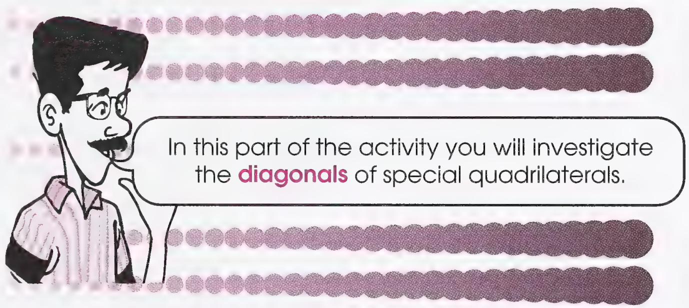
12. For each of the following parallelograms, calculate the unknown angles. Do **not** use a protractor.

a. 

b. 

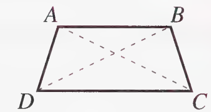
Check your answers by turning to the Appendix.

Investigating Diagonals of Special Quadrilaterals

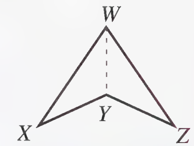


A diagonal is a line segment joining any two vertices of a polygon not already joined.

Many quadrilaterals have two diagonals. For example, in this quadrilateral, line segments AC and BD are diagonals.



Some quadrilaterals have only one diagonal. For example, in this quadrilateral, line segment WY is a diagonal.



13. Copy the following table.

Name of Quadrilateral	Are the Two Diagonals of Equal Length?	Do the Two Diagonals Bisect Each Other?	Do the Two Diagonals Cross at Right Angles?
Parallelogram			
Rhombus			
Rectangle			
Square			
Trapezoid			
Isosceles Trapezoid			
Trapezium			
Kite			
Dart			

Find the sheet of quadrilaterals in the Appendix. Photocopy the page (or trace the quadrilaterals). On the photocopy (or the tracing) draw the diagonals of each figure. Measure the diagonals, the line segments formed by the intersection of the diagonals, and the angles where the diagonals meet. Record these measurements. **Hint:** When measuring angles, you may find it helpful to extend the diagonals past the confines of the figures.

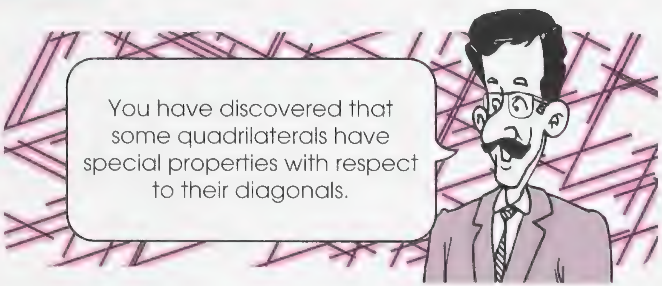
Use your findings to complete the given table. If a figure does not have two diagonals, write N/A.

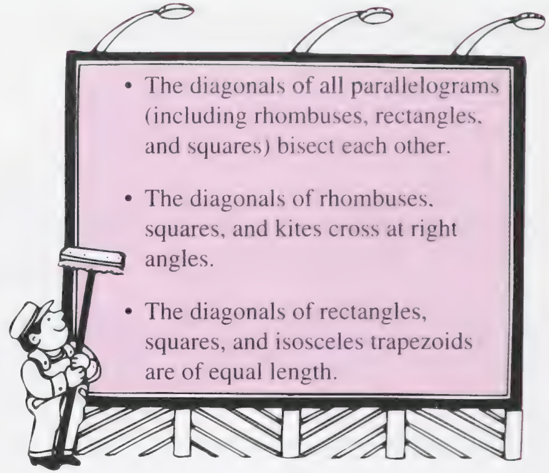
14. **a.** In which quadrilaterals are the diagonals of equal length?

b. In which quadrilaterals do the diagonals bisect each other?

c. In which quadrilaterals do the diagonals cross at right angles?

Check your answers by turning to the Appendix.

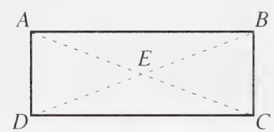




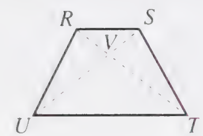
Use these properties to answer questions 15 and 16. Do **not** measure the diagonals or the angles.

15. In each of the following figures, name the pairs of segments that have the same measurement.

a. Rectangle *ABCD*

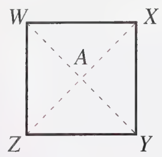


b. Isosceles Trapezoid *RSTU*

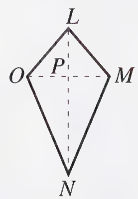


16. Name the right angles in each of the following figures.

a. Square *WXYZ*

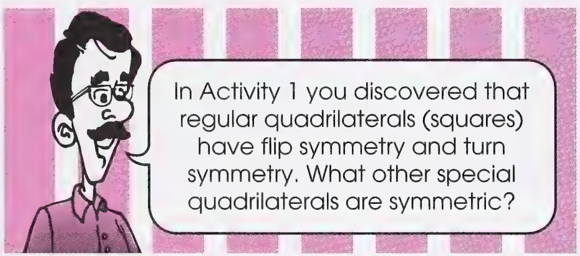


b. Kite *LMNO*



Check your answers by turning to the Appendix.

Investigating Symmetry in Special Quadrilaterals



17. Find the sheet of quadrilaterals in the Appendix. Test each of the figures for flip symmetry and turn symmetry. **Note:** You will need tracing paper, scissors, and a pin.

Use your findings to complete a table like this. **Note:** If a figure does not have flip symmetry or turn symmetry, write N/A.

Name of Quadrilateral	Number of Diagonal Lines of Symmetry	Number of Non-Diagonal Lines of Symmetry	Turn Order
Parallelogram			
Rhombus			
Rectangle			
Square			
Trapezoid			
Isosceles Trapezoid			
Trapezium			
Kite			
Dart			

Check your answers by turning to the Appendix.

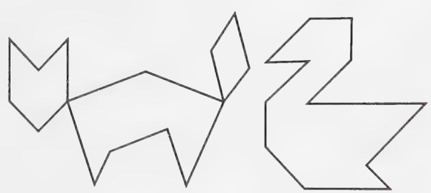
Now Try This

Use a problem-solving strategy to answer the following question.

Find the tangram puzzle in the Appendix. Photocopy the tangram and glue it to heavy paper or card stock; then cut out the pieces.

The tangram is an ancient Chinese puzzle that has seven geometric shapes called **tans** (two large triangles, one medium triangle, two small triangles, a square, and a parallelogram).

The object of tangram puzzles is to form shapes like these using all seven tans.



When forming a shape, you may have to flip one or more of the tans.

18. **a.** In the Appendix, find Tangram Puzzle 1. Use all seven tans to form the rectangle. **Hint:** Lay the tans on the puzzle.

b. In the Appendix, find Tangram Puzzle 2. Use all seven tans to form the parallelogram. **Hint:** Lay the tans on the puzzle.

c. In the Appendix, find Tangram Puzzle 3. Use all seven tans to form the pentagon. **Hint:** Lay the tans on the puzzle.

d. In the Appendix, find Tangram Puzzle 4. Use all seven tans to form the hexagon. **Hint:** Lay the tans on the puzzle.

Check your answers by turning to the Appendix.

Use the search engines on the Internet to explore *tangrams*. There are many sites with various examples of tangrams. There are also sites that allow you to manipulate tans.

Activity 4: Networks

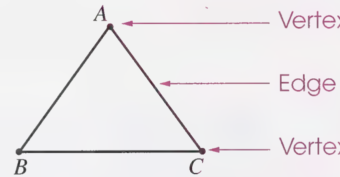
PHOTO SEARCH LTD.

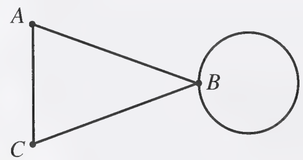
What do you think of when you hear the word *network*? Do you think of the network of roads connecting all the houses and businesses in your community?

Perhaps you think of a network of television or radio stations, such as the CBC, or a network of computers, such as the Internet.

In mathematics a **network** is a figure consisting of **edges** and **vertices**. A network may also be called a **graph**; however, a network bears no relation to a graph which charts data.

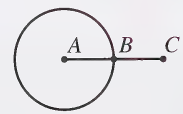
This is a network; it has 3 edges and 3 vertices. The vertices are at the junctions of the edges.

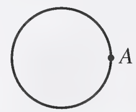




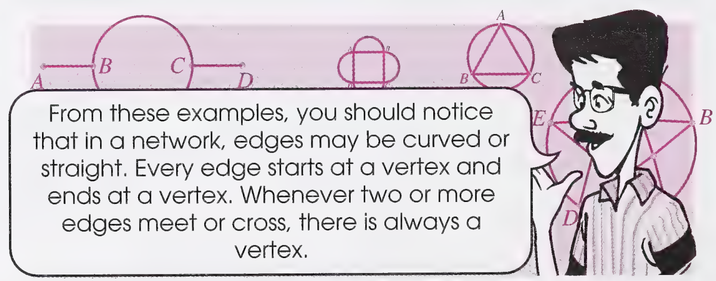
This is a network; it has 4 edges and 3 vertices. The vertices are at the junctions of the edges.

This is a network; it has 3 edges and 3 vertices. The edges each begin and end with a vertex.

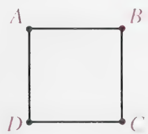


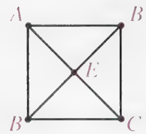


This is a network; it has 1 edge and 1 vertex. The edge begins and ends with a vertex.

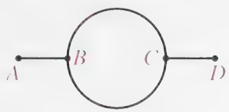


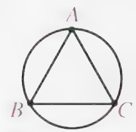
1. How many vertices and how many edges do each of the following networks have?

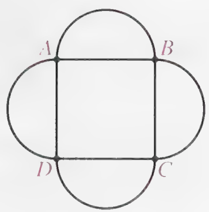
a. 

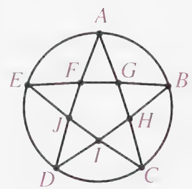
b. 

c. 

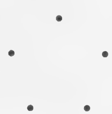
d. 

e. 

f. 

g. 

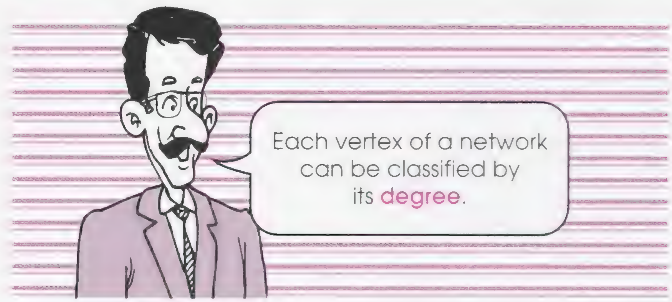
2. Place a sheet of lightweight plain paper over the following set of vertices and use the set of vertices to make each of the following networks.



Note: Remember that when two edges cross, there will be a vertex. Do not create any new vertices.

a. a network with 5 vertices and 4 edges
b. a network with 5 vertices and 5 edges
c. a network with 5 vertices and 6 edges
d. a network with 5 vertices and 7 edges
e. a network with 5 vertices and 8 edges
f. a network with 5 vertices and 9 edges

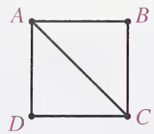
Check your answers by turning to the Appendix.



The degree of a vertex is the number of edges that meet it.

When an odd number of edges meet a vertex, the vertex is called an **odd vertex**. When an even number of edges meet a vertex, the vertex is called an **even vertex**.

For example, examine the following network.



A is an odd vertex because its degree is 3; three edges meet A.

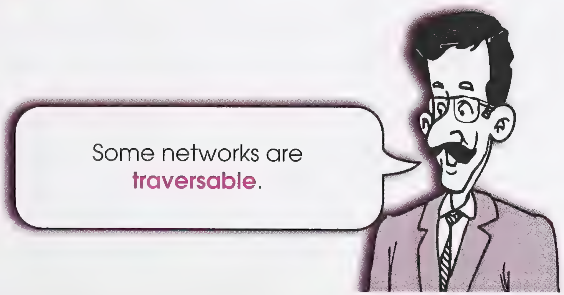
B is an even vertex because its degree is 2; two edges meet B.

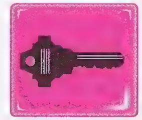
C is an odd vertex because its degree is 3; three edges meet C.

D is an even vertex because its degree is 2; two edges meet D.

3. For each network in question 1, state the degree of vertex B. For each network state whether vertex B is an even or odd vertex.

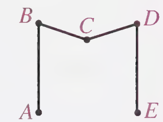
Check your answers by turning to the Appendix.

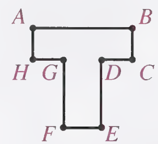




A network is traversable if it has a path that travels along every edge exactly once. (The path may pass through individual vertices more than once).

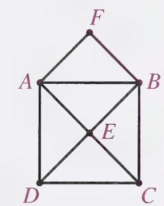
This network is traversable. One path passes through A, B, C, D, E.

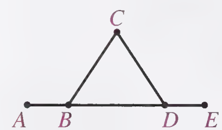




This network is traversable. One path passes through $A, B, C, D, E, F, G, H, A$.

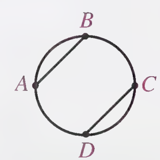
This network is traversable. One path passes through $D, A, F, B, C, E, A, B, E, D, C$. **Note:** The network can only be traversed starting at D or C.





This network is **not** traversable.

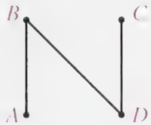
This network is **not** traversable.

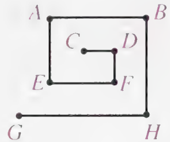


4. Each of the following networks is a simple open figure. Is each traversable? Answer **yes** or **no**. If yes, give the path.

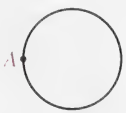
a. 

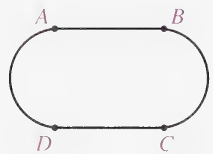
b. 

c. 

d. 

5. Each of the following networks is a simple closed figure. Is each traversable? Answer **yes** or **no**. If yes, give the path.

a. 

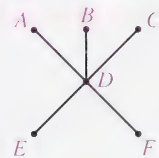
b. 

6. a. Based on the examples in question 4, does it seem that every network that is a simple open figure is traversable?

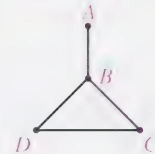
b. Based on the examples in question 5, does it seem that every network that is a simple closed figure is traversable?

7. Place a sheet of lightweight plain paper over each of the following networks and try to trace it without retracing any edge or taking your pencil off the paper. Keep a record of the networks that can be traversed and the paths. **Hint:** Some of the networks can only be traversed if you start from certain vertices.

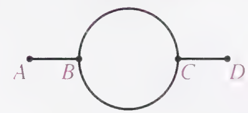
Network 1



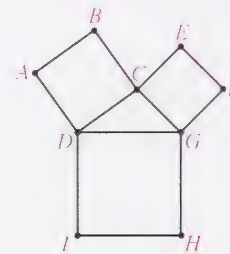
Network 2



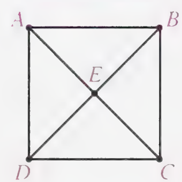
Network 3



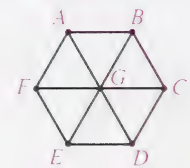
Network 4



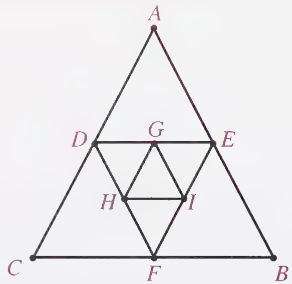
Network 5



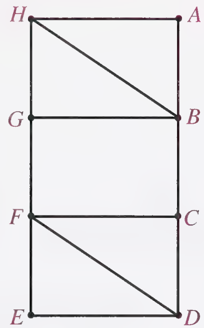
Network 6



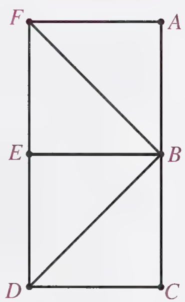
Network 7



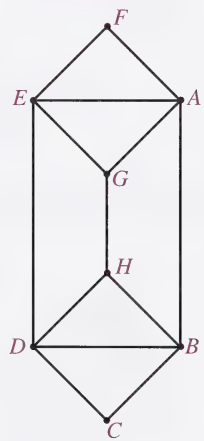
Network 8



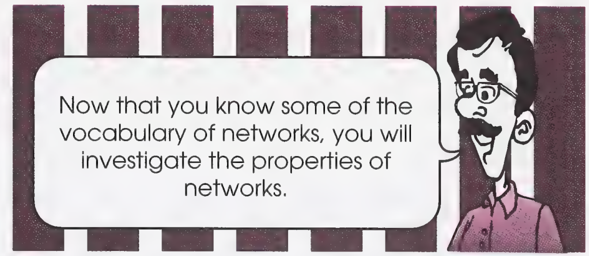
Network 9



Network10



Check your answers by turning to the Appendix.



8. **a.** Copy each of the networks in question 7. Beside each vertex indicate its degree.

b. Use your findings in question 8.a. to complete a table like the following.

Network	Number of Edges	Sum of the Degrees of the Vertices
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

c. What do you notice about the relationship between the sum of the degrees of the vertices and the number of edges in each network?

9. a. Use your findings in questions 7 and 8 to complete a table like the following.

Network	Number of Even Vertices	Number of Odd Vertices	Can the Network be Traversed?
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			

b. Does it seem that a network can be traversed if none of the vertices are odd?

c. Does it seem that a network can be traversed if it has two odd vertices?

d. Does it seem that a network can be traversed if it has more than two odd vertices?

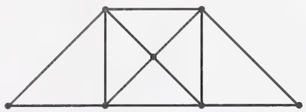
e. Networks 2 and 10 have two odd vertices and are traversable. Examine the paths by which these networks can be traversed. What do you notice?

Check your answers by turning to the Appendix.

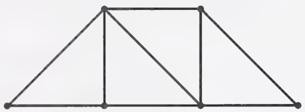
- The sum of the degrees of the vertices of a network is twice the number of edges.
- If a network has 0 or 2 odd vertices, it can be traversed. If a network has more than 2 odd vertices, it cannot be traversed.
- If a network has no odd vertices, it can be traversed beginning at any vertex.
- If a network has 2 odd vertices, it can be traversed by beginning at one of the odd vertices and ending at the other.

10. For each of the following networks, state whether or not the network can be traversed and give a reason why or why not.

a.



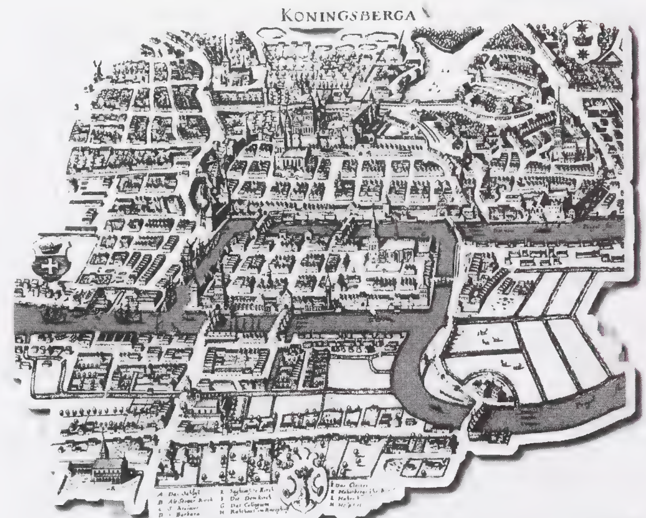
b.



Check your answers by turning to the Appendix.

One of the problems that led to the development of network theory involves the seven bridges that made it possible to travel from one part of the city of Königsberg (in old Germany) to another.

The following drawing from a book published about 1650 shows the seven bridges of Königsberg.

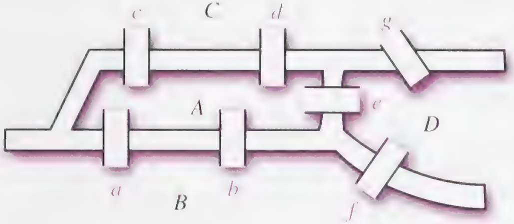


After *Topographia Prussiae et Pomerelliae* by M. Zeiller Frankfurt, c.1650.

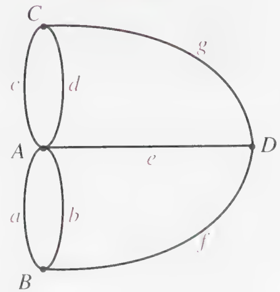
The citizens of Königsberg wondered if it was possible to take a round-trip walk in which each of the seven bridges was crossed only once. Everyone who tried it ended up either skipping a bridge or recrossing at least one bridge.

Most mathematicians who studied this problem thought that the round-trip walk was not possible, but they could not prove this theory.

Eventually the problem of the Königsberg bridges came to the attention of the Swiss mathematician Leonard Euler. In an article published in 1736, Euler used network theory to prove that the round-trip walk was not possible. Euler began by drawing a map. He labelled the four parts of the city as A, B, C, and D. He labelled the seven bridges as a, b, c, d, e, f, and g.



Euler then simplified the map further by using a network. He represented the four parts of the city by four vertices. He represented the seven bridges by seven edges.



Because this network has four odd vertices, it cannot be traversed. This proves that the seven bridges of Königsberg could not be travelled in one round-trip walk.

11. In 1875, an eighth bridge was built connecting areas B and C.

a. Revise the network to include this eighth bridge. Label the bridge h.

b. Was it now possible to make a round-trip walk crossing all eight bridges only once? Explain.

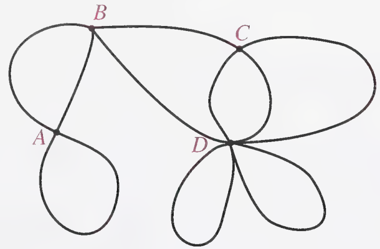
Check your answers by turning to the Appendix.

Use the Internet to discover more about networks (graphs). Following is the uniform resource locator of a site that you may find interesting.

http://www.c3.lanl.gov/mega-math/workbk/graph/graph.html

Network theory has many practical applications. It is used in designing the electrical circuits in computers and small appliances, channelling information in large organizations, and designing efficient bus routes.

12. The following network shows a map of a park. The vertices represent rest stops. The edges represent trails.



a. Copy and complete the following table to show the number of trails from one rest stop to a second rest stop. Do not count trails that pass through another rest stop. **Note:** Two trails lead from A to A, one in a clockwise direction and one in a counterclockwise direction.

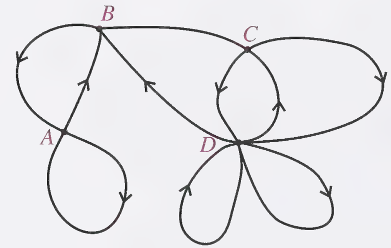
To / From	A	B	C	D
A	2	2	0	0
B				
C				
D				

b. Find the sum of the first row in the table. What does this represent on the map?

c. Find the sum of the fourth column in the table. What does this represent on the map?

d. Count the number of trails on the map. Find the sum of all the numbers in the table. How is this sum related to the number of trails on the map? Why?

13. The following network shows a map of the park (in question 12) after the park rangers designated some ecologically sensitive trails as one-way trails. **Note:** Each arrow on the map indicates the direction in which travel is permitted on the trail. Edges without an arrow indicate two-way trails.



a. Copy and complete the following table to show the number of trails from one rest stop to a second rest stop. Do not count trails that pass through another rest stop.

To / From	A	B	C	D
A	1	1	0	0
B				
C				
D				

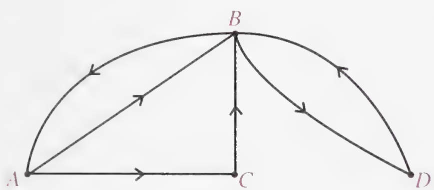
b. Find the sum of the numbers in the first row in the table. What does this represent on the map?

c. Find the sum of the numbers in the fourth column in the table. What does this represent on the map?

d. Find the sum of all the numbers in the table. How is this total related to the number of edges in the network? Why?

14. Network theory is used to design efficient air-travel routes.

The following network shows a map of the air-travel routes between four cities.



The arrows on the map indicate the direction in which travel can take place.

a. Copy and complete the following table to show the number of trips with no stopovers from each city. **Hint:** There is no way to get from A to D without a stopover.

Table 1: No Stopovers

From \ To	A	B	C	D
A				
B				
C				
D				

b. Copy and complete the following table to show the number of trips with one stopover from each city.

Table 2: One Stopover

From \ To	A	B	C	D
A				
B				
C				
D				

c. Copy and complete the following table to show the number of trips from each city with, at most, one stopover.

Table 3: No Stopover or One Stopover

From \ To	A	B	C	D
A				
B				
C				
D				

d. How is Table 3 related to Table 1 and Table 2?

e. Can you travel from one city to every other city in the network with, at most, one stopover? Explain.

Check your answers by turning to the Appendix.

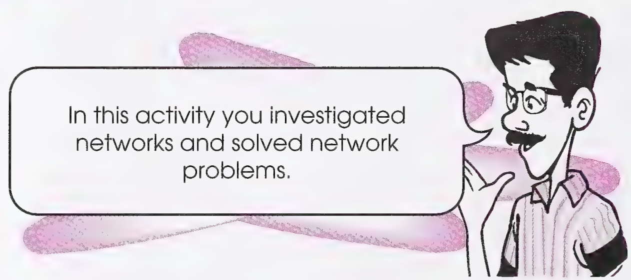
Now Try This

Use a problem-solving strategy to answer the following question.

15. The servers at the Corner Cafe put their tips in glasses behind the front counter. Each tip was placed in the appropriate glass. Alice put $2 in her glass and $1 in Dawn's glass. Barb put $1 in Alice's glass and $3 in Cathy's glass. Cathy put $1 in her glass, $1 in Alice's glass, $2 in Barb's glass, and $2 in Edwina's glass. Dawn put $2 in Cathy's glass. Edwina put $2 in Alice's glass and $1 in Dawn's glass.

    Find the amount of tips each server earned.

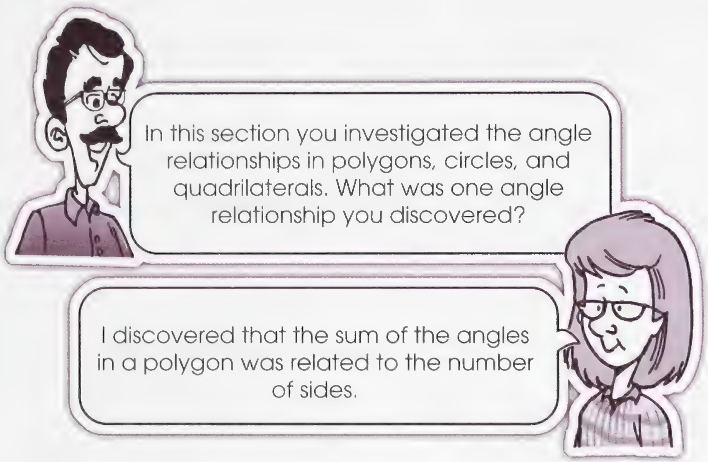
Check your answer by turning to the Appendix.



Follow-up Activities

If you had difficulties understanding the concepts and skills in the activities, it is recommended that you do the Extra Help. If you have a clear understanding of the concepts and skills, it is recommended that you do the Enrichment. You may decide to do both.

Extra Help



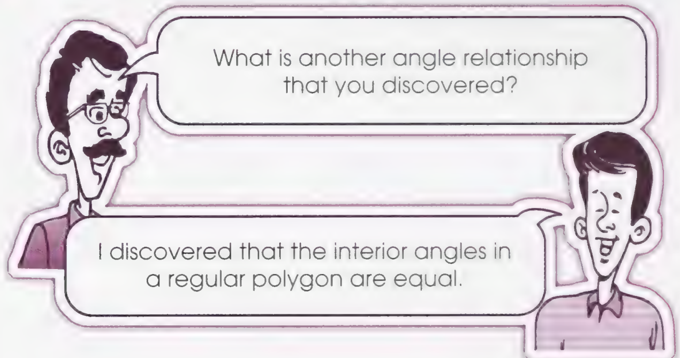
The relationship between the sum of the angles and the number of sides in a polygon can be described by the following equation, where s is the sum of the interior angles (in degrees) and n is the number of sides.

$$s = (n - 2)180°$$

Use this angle relationship to answer question 1.

1. **a.** What is the sum of the angles in a quadrilateral?
   b. What is the sum of the angles in an octagon?
   c. What is the sum of the angles in a dodecagon?

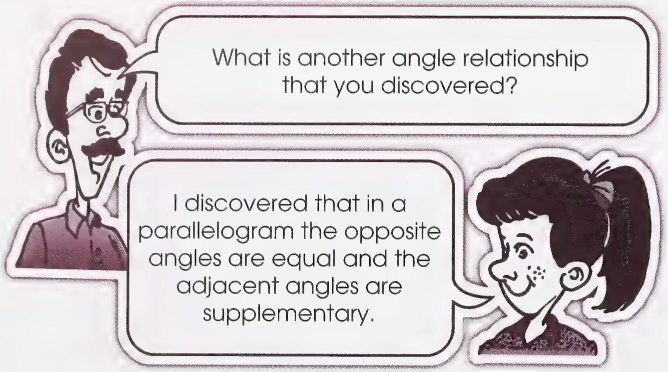
Check your answers by turning to the Appendix.



Use this angle relationship to answer question 2.

2. **a.** What is the measure of each angle in a square?
   b. What is the measure of each angle in a regular octagon?
   c. What is the measure of each angle in a regular dodecagon?

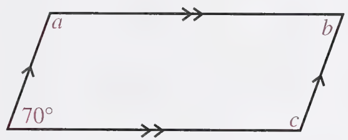
Check your answers by turning to the Appendix.



Use this angle relationship to answer question 3.

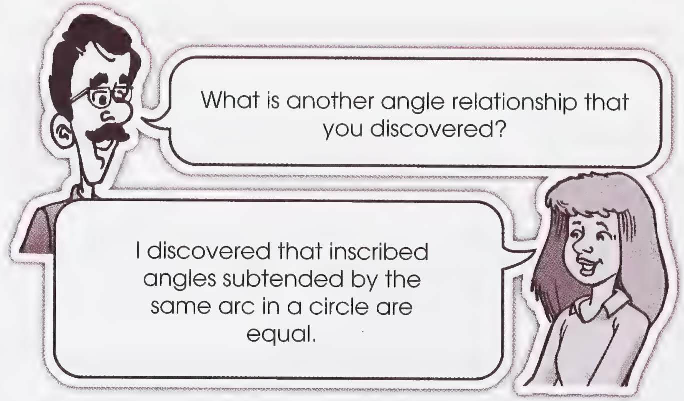
3. Calculate the missing angles in each of the following parallelograms. **Note:** Do **not** measure the angles.

a.



b.

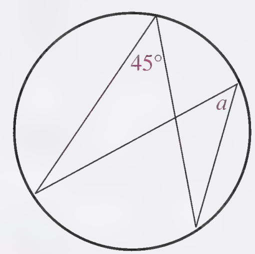
Check your answers by turning to the Appendix.



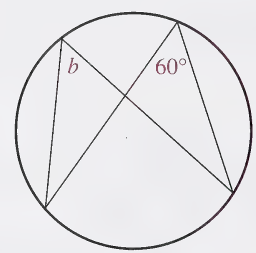
Use this angle relationship to answer question 4.

4. Calculate the missing angle in each of the following diagrams. **Note:** Do **not** measure the angles.

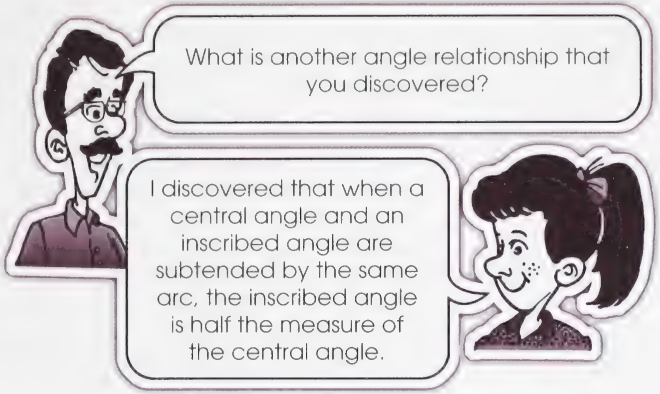
a.



b.



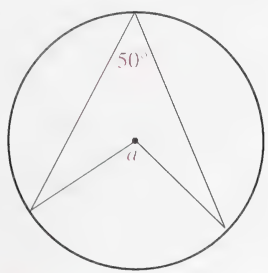
Check your answers by turning to the Appendix.



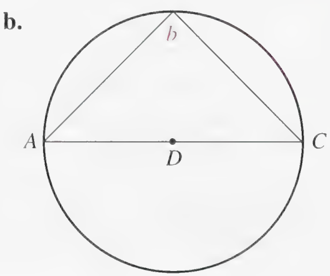
Use this angle relationship to answer question 5.

5. Calculate the measure of the missing angle in each of the following diagrams. **Note:** Do **not** measure the angles.

a.

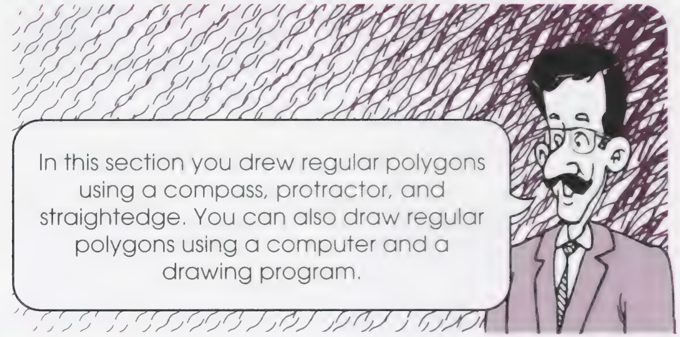


b.



Check your answers by turning to the Appendix.

Enrichment



One computer drawing program that many schools use is **Logo**.

Dr. Seymour Papert, working with a group of scientists, created Logo during the late 1960s and early 1970s. Dr. Papert got the idea for this computer program after watching a computer direct a pen to draw a picture. The pen was mounted in an apparatus that looked like a turtle.

If you wish to know more about Dr. Papert and Logo programming, you may find this site interesting. It features questions and answers on Logo, a glossary, references, programs you can download, and more.

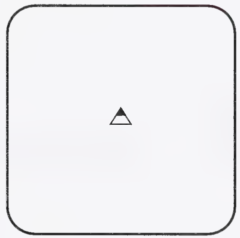
http://www.primenet.com/pcai/New_Home_Page/ai_info/pcai_logo.html

Schools in your district may also have Logo programs such as *Logo Plus™ for the Macintosh* or *PC Logo* from Terrapin Software, Inc. There are many distributors of Logo programs.

Following are the steps to draw a square with sides of 30 units.

Step 1: Load Logo in the computer.

When Logo is loaded into a computer, the **turtle** (a triangular shaped drawing tool) is active and appears in the turtle window. The turtle is in its **home** position.



Step 2: To draw the square, enter one of the following series of commands. Press "Return" after each command.

FORWARD 30	**or**	FD 30
RIGHT 90		RT 90
FORWARD 30		FD 30
RIGHT 90		RT 90
FORWARD 30		FD 30
RIGHT 90		RT 90
FORWARD 30		FD 30
RIGHT 90		RT 90

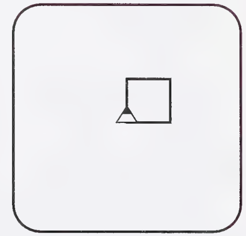
Alternatively, you can enter the following command:

REPEAT 4 [FORWARD 30 RIGHT 90]

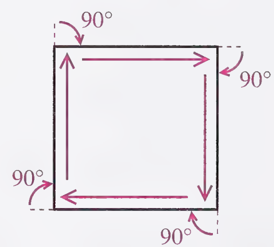
or

REPEAT 4 [FD 30 RT 90]

The turtle draws the square and returns to the home position. It is in the centre of the screen pointing in an upwards direction.



Notice that the turtle moves forward 30 steps, turns right 90°, and repeats this sequence a total of four times. The turtle makes a trip of 360° around the screen.



$4 \times 90° = 360°$

Note: To erase a drawing, enter the command **CLEARSCREEN** or **CS** and press "Return."

Use a computer and a Logo program to answer questions 1 and 2.

1. What commands would you enter to draw the following regular polygons? (**Note:** Enter the commands if you have access to a computer and a Logo program.)

   a. a regular triangle with sides of 40 units
   b. a regular pentagon with sides of 40 units
   c. a regular hexagon with sides of 30 units
   d. a regular octagon with sides of 30 units

2. Draw a regular polygon with 40 sides of 1 unit by entering the following command and pressing "Return."

   ```
   REPEAT 40 [FORWARD 1 RIGHT 9]
   ```

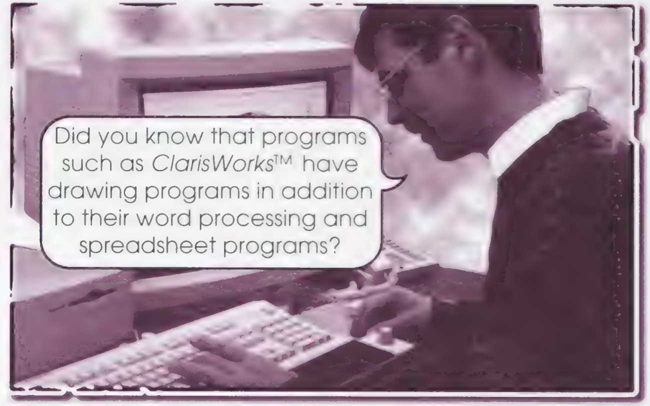
   or

   ```
   REPEAT 40 [FD 1 RT 9]
   ```

   What shape does this regular polygon resemble? **Note:** Increase the side length. What happens?

Check your answers by turning to the Appendix.

Many books are available on Logo. Your local library may have such books available. You may wish to use one of these books to learn more about Logo.



Here are the steps to draw a regular polygon using *ClarisWorks*™.

Step 1: Start a new drawing document from the New Document dialogue box that appears when you first start *ClarisWorks*™.

Step 2: Select the regular polygon tool. It looks like this. ◇

Step 3: Choose "Polygon Sides" from the Options menu, type the number of sides in the regular polygon, and click "OK."

Step 4: Use the mouse to position the pointer where you want the edge of the regular polygon to appear and drag to a point where you want a corner of the regular polygon to appear. When the regular polygon is the size you want, release the mouse button.

You may wish to experiment with a drawing program. The *User's Guide* will give you more information.

Conclusion

In this section you investigated the properties of quadrilaterals, polygons, and circles. You solved network and colour problems

What shapes do you see in this photograph of Keltic Lodge in Cape Breton, Nova Scotia? Do you see a trapezoid, a triangle, a square, a rectangle, and an arc of a circle?

Many two-dimensional shapes are used in architectural designs. Take a look around your neighbourhood. What types of shapes can you see?

Assignment

Assignment Booklet

You are now ready to complete the assignment for Section 1.

Section 2: Measurement

PHOTO SEARCH LTD

Have you ever used snowshoes? Snowshoes allow you to walk more easily through deep snow. Snowshoe racers, moving with the characteristic bent-knee, shuffling gait, can cover 1.6 km in about 5 min.

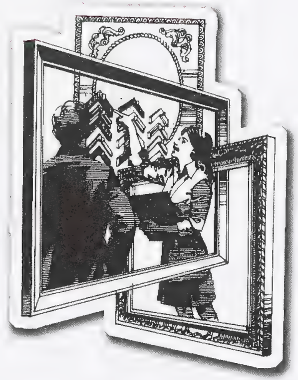
How is this possible? The reason a person with snowshoes sinks less in deep snow is that the area of each snowshoe is much greater than the area of the sole of a boot. The person's mass is therefore distributed over a greater area and the person sinks less.

In this section you will generalize measurement patterns and procedures and solve problems involving perimeter and area.

Activity 1: Perimeter and Circumference

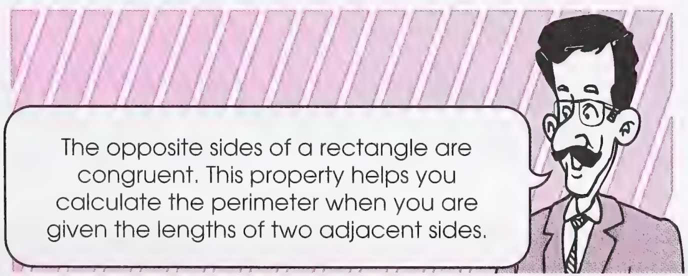
To find out how much material you need to frame a rectangular photograph, you need to find the **perimeter** of the photograph.

To find out how many bricks you need to edge a circular garden, you need to find the **circumference** of the garden.



Perimeter is the distance around a figure. The distance around a circle is called the circumference.

Perimeter of a Rectangle



You can find the perimeter (P) of any rectangle by doubling the length (ℓ), doubling the width (w), and then adding the two products.

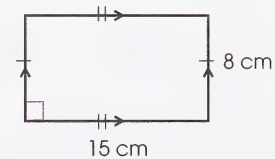
This rule can be expressed by the following formula.

$$P = 2\ell + 2w$$

You can use this formula to find the perimeter of any rectangle.

Example

Find the perimeter of this rectangle.



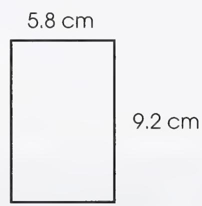
Solution

$$\begin{aligned} P &= 2\ell + 2w \\ &= 2(15) + 2(8) \\ &= 30 + 16 \\ &= 46 \end{aligned}$$

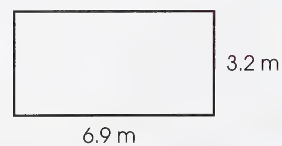
The perimeter of the rectangle is 46 cm.

1. Calculate the perimeter of each of the following rectangles.

a.



b.

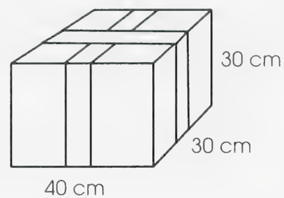


2. Anthony is putting new baseboards along the edge of the floor in a rectangular room that is 4 m long and 3 m wide.

   a. What is the perimeter of the room?

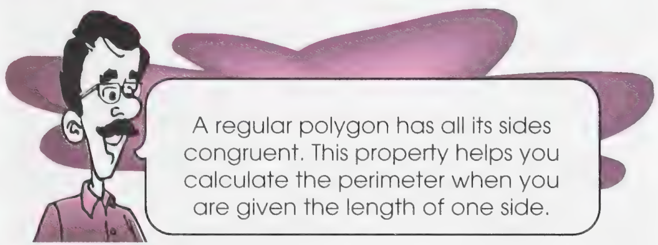
   b. If baseboards cost \$4.79/m, how much will the baseboards cost?

3. George used tape to seal a box. He wound the tape around the box in two directions. What was the smallest length of tape that George could have used?



Check your answers by turning to the Appendix.

Perimeter of a Regular Polygon



You can find the perimeter (P) of any regular polygon by multiplying the number of equal sides (n) by the length of one side (s).

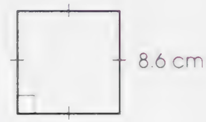
This rule can be expressed by the following formula.

$$P = ns$$

You can use this formula to find the perimeter of any regular polygon.

Example 1

Find the perimeter of this square.



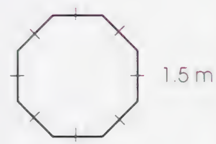
Solution

$$\begin{aligned} P &= ns \\ &= 4(8.6) \\ &= 34.4 \end{aligned}$$

The perimeter of the square is 34.4 cm.

Example 2

Find the perimeter of this regular octagon.



Solution

$$\begin{aligned} P &= ns \\ &= 8(1.5) \\ &= 12 \end{aligned}$$

The perimeter of the regular octagon is 12 m.

4. The Pentagon near Washington, D.C., is so named because of its shape.

COURTESY OF THE U.S. DEPARTMENT OF DEFENSE

Each of the outer walls of the Pentagon is 302 m long. Find the minimum distance (in metres) that a person would travel while walking around the outside of the Pentagon.

5. A farmer wishes to put a fence with three strands of barbwire around his pasture. The pasture is a square which is 820 m on a side.

   a. What is the perimeter of the field?
   b. What length of barbwire will the farmer have to purchase?

6. The first Ferris wheel was built by George Ferris for the World's Columbia Exposition in Chicago in 1893. It was a regular polygon with 36 sides. Each side was 6.6 m long.

What was the perimeter of the first Ferris wheel?

Check your answers by turning to the Appendix.

INTERNET

You may wish to find out more interesting facts about the first Ferris wheel. Use your search engines to explore *Ferris wheel.*

Circumference of a Circle

The ratio of the circumference of a circle to the diameter of the circle is the same for any circle, regardless of its size. This ratio is a constant value and is represented by the Greek letter π (pi).

This property helps you calculate the circumference when you are given the diameter of the circle.

This equation shows that the ratio of the circumference to the diameter is equal to π.

$$\frac{C}{d} = \pi$$

If you multiply each side of the equation by d, you get the formula for the circumference of a circle.

$$d\left(\frac{c}{d}\right) = \pi d$$
$$C = \pi d$$

Because the diameter is twice the radius, the following formula may also be used to calculate the circumference of circle.

$$C = 2\pi r$$

You can use either of these formulas to find the circumference of any circle. Remember, π is about 3.14. **Note:** Some calculators have a key for π.

Example

Find the circumference of this circle, to the nearest centimetre.

Solution

$C = \pi d$

$\doteq 3.14(8)$ ← Use a calculator; you may press the π key instead of entering 3.14.

$\doteq 25$ ← Round to the nearest centimetre.

The circumference of the circle is about 25 cm.

7. The diameter of the clock face on the Big Ben tower in London, England, is 7.1 m. Find the circumference of the clock face, to the nearest tenth of a metre.

8. The diameter of Earth is approximately 12 750 km.

a. What is the circumference of Earth, to the nearest kilometre?

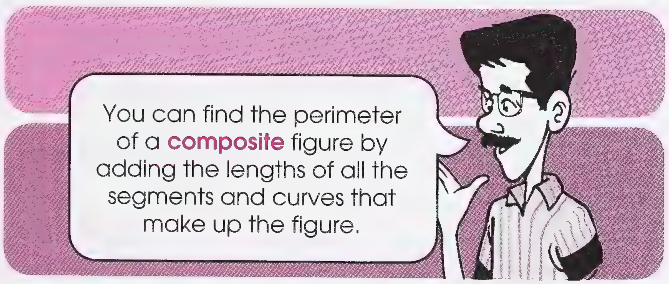
b. A satellite is orbiting Earth 36 000 km above Earth's surface. How far does the satellite travel in completing one orbit? Round the answer to the nearest kilometre.

9. The diameter of a basketball is 24.5 cm. The diameter of a basketball hoop is 45 cm.

How much greater is the circumference of the hoop than the circumference of the basketball? Round the answer to the nearest centimetre.

Check your answers by turning to the Appendix.

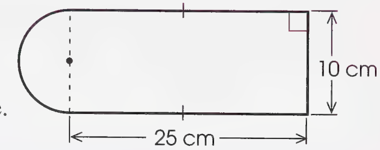
Perimeter of a Composite Figure



A composite figure is a figure made up of two or more shapes.

Example

Find the perimeter of this figure, to the nearest centimetre.



Solution

Step 1: Examine the figure to determine what segments and curves make up the perimeter.

The perimeter of this figure is made up of a semicircle with a diameter of 10 cm, two segments of 25 cm, and one segment of 10 cm.

Step 2: Calculate the circumference of the semicircle; it is half the length of the circumference of a circle with the same diameter.

$$\begin{aligned} C &= \pi d \\ &\doteq 3.14(10) \\ &\doteq 31.4 \end{aligned} \qquad \frac{1}{2}(31.4) = 15.7$$

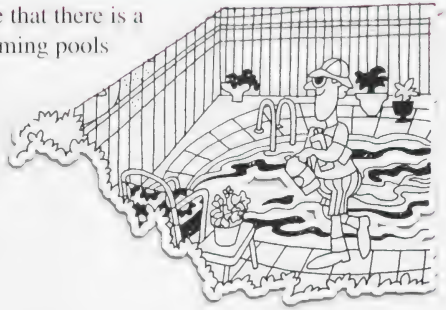
The circumference of the semicircle is about 15.7 cm.

Step 3: Calculate the perimeter of the figure and round.

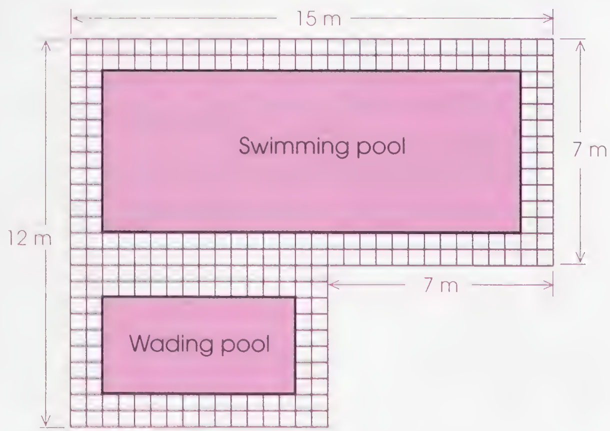
$$\begin{aligned} P &\doteq 15.7 + 2(25) + 10 \\ &\doteq 15.7 + 50 + 10 \\ &\doteq 75.7 \leftarrow \text{Round to the nearest centimetre.} \\ &\doteq 76 \end{aligned}$$

The perimeter of the figure is about 76 cm.

10. Regulations require that there is a fence around swimming pools and wading pools.



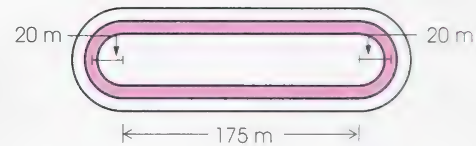
a. If a fence is put around the outside of this swimming pool and wading pool, what length of fencing is required?



b. If a fence is also placed between the swimming pool and the wading pool, what length of fencing is needed altogether?

11. Joe enjoys competing in races.

a. If Joe stays in the middle of the inside lane of the track shown in the diagram, how far does he travel in one lap? Round to the nearest metre.



b. If Joe stays in the middle of the outside lane, how far does he travel in one lap? Round to the nearest metre.

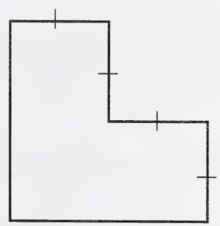
Check your answers by turning to the Appendix.

Now Try This

Use a problem-solving strategy to answer the following questions.

12. A spider is at the bottom of a well that is 8 m deep. If the spider climbs up 4 m each day, but slips down 3 m each night, how many days will it take the spider to reach the top of the well?

13. A developer has bought this L-shaped piece of land. The developer wants to subdivide it into four congruent lots. How can this be done?



14. Ace Vending Company has a quick way to find the value of a pile of coins. They weigh them!

This pile of dimes has a mass of 327.06 g. If one dime has a mass of 2.07 g, what is the value of the pile of coins?

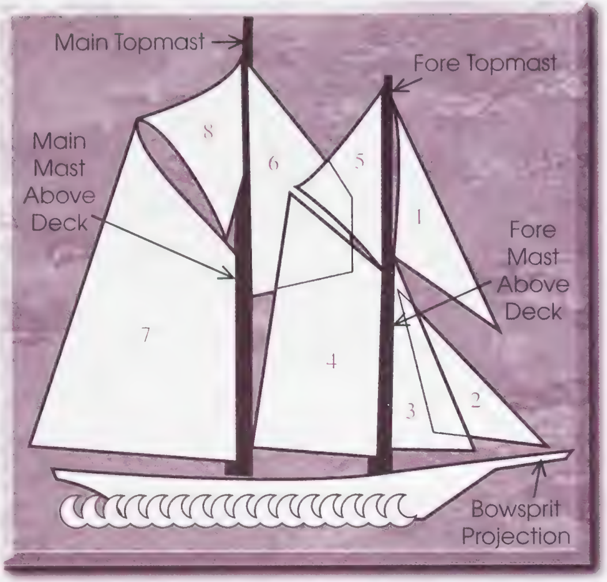
Check your answers by turning to the Appendix.

Did You Know?

The *Bluenose*, a symbol of national pride, is featured on Canadian dimes. Find the article entitled "The Fishing Schooner *Bluenose*" in the Appendix. Read the article and then answer the following questions.

15. a. Who designed the Bluenose? When was it built?
    b. Why is the *Bluenose* a symbol of Canadian pride?

16. Use this diagram of the *Bluenose* to answer the following questions.



1	Jib topsail	5	Fore gaff-topsail
2	Jib	6	Fisherman staysail
3	Jumbo (fore staysail)	7	Mainsail
4	Foresail	8	Main gaff-topsail

a. What shape does the foresail (4) resemble?
b. What shape does the jib topsail (1) resemble?

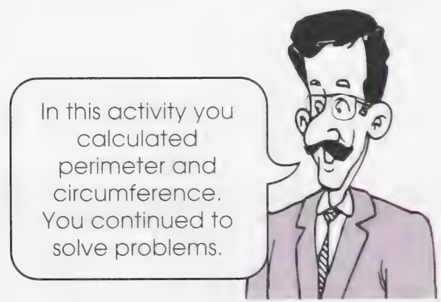
17. The *Bluenose* is pictured on the face of every Canadian dime. What is the circumference of the face of a dime? Round to the nearest tenth of a centimetre. **Hint:** The diameter is 1.8 cm.

Check your answers by turning to the Appendix.

Use the Internet to find more information on the *Bluenose*. You may find this uniform resource locator (URL) useful.

http://www.cs.ubc.ca/spider/flinn/bluenose/bluenose.html

You may use your search engines to explore many other sites.



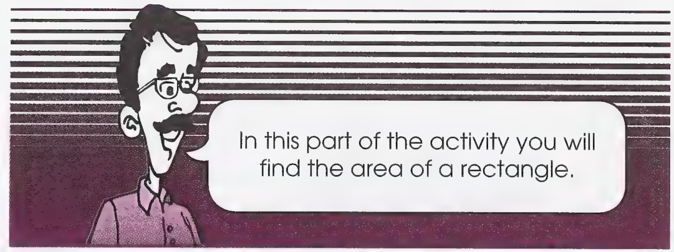
Activity 2: Area

Area is used in many everyday situations.

When you buy carpet for the floor, paint for the walls, or blinds for the windows, you consider area.

When you apply fertilizer or herbicides to the lawn, you use area.

Area of a Rectangle



1. Find the page in the Appendix with a rectangle drawn on 1-cm grid paper. Find the area of this rectangle.

Check your answer by turning to the Appendix.

When you answered question 1, you probably did not count all the square units to find the area of the rectangle. You probably counted the number of square units along the length and multiplied this number by the number of square units along the width.

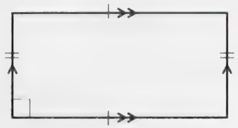
This rule can be expressed by the following formula, where A is the area in square units, ℓ is the length in linear units, and w is the width in linear units.

$$A = \ell w$$

You can use this formula to find the area of any rectangle.

Example

Find the area of this rectangle.



Solution

$A = \ell w$

$= 32.5(12.2)$ ← Use a calculator to multiply.

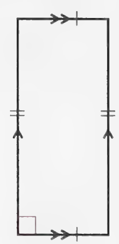
$= 396.5$

The area is $396.5\ \text{cm}^2$.

2. Use the formula to find the area of each rectangle.

a.

b.



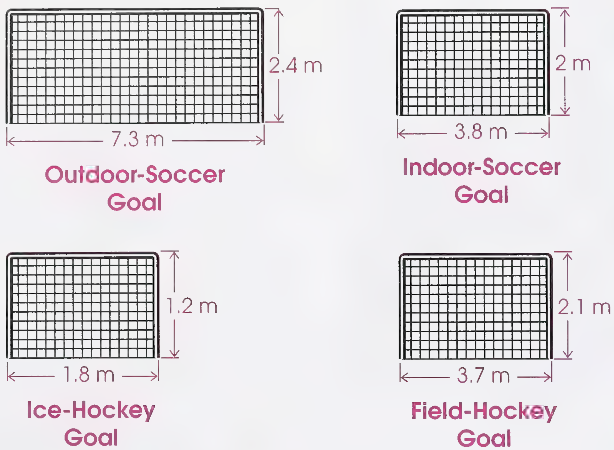
3. Margaret rototills a rectangular garden that is 15 m by 12 m. What is the area that she rototills?

4. An average hockey rink is about 60.6 m long and 26 m wide. An Olympic hockey rink is about 60.6 m long and 30.3 m wide.

About how much greater an area does an Olympic hockey rink cover? (**Note:** Ice rinks have rounded corners; however, for this question, rinks are considered rectangular.)

5. A rectangular backyard that is 25 m × 30 m is to be seeded with grass seed. If 1 kg of grass seed will cover $200\ \text{m}^2$, how much grass seed is needed?

6. The following diagrams show the openings of different kinds of goals. **Note:** The diagrams are not drawn to scale.

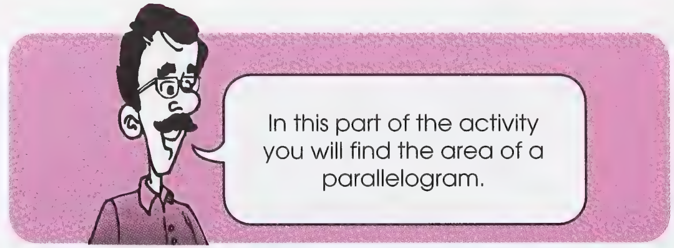


a. How much greater is the opening of the outdoor-soccer goal than the indoor-soccer goal?

b. How much greater is the opening of the field-hockey goal than the ice-hockey goal?

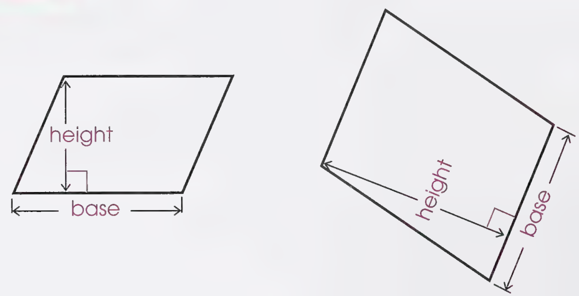
Check your answers by turning to the Appendix.

Area of a Parallelogram



Before beginning your investigation of the area of a parallelogram, you need to know two terms—**base** and **height**.

Any side of a parallelogram is the base. The perpendicular distance from the base to the opposite side is the height.



7. **a.** Find the page in the Appendix with a parallelogram drawn on 1-cm grid paper. Give the measurements of the base and height of the parallelogram.

b. Make a photocopy of the page. Cut out the parallelogram. Then cut off a triangle along the indicated grid line. Tape the two parts together to form a rectangle.

Give the measurements of the length and width of the new rectangle. How do the measurements compare to the base and height of the parallelogram?

c. Determine the area of the rectangle.

d. What can you conclude about the area of the parallelogram?

Check your answers by turning to the Appendix.

You have discovered that a parallelogram can be made into a rectangle by cutting off a triangular end and taping this triangle to the opposite side.

The length of the new rectangle equals the length of the base of the parallelogram and the width of the new rectangle equals the height of the parallelogram. Because of this, the area of a parallelogram can be found by multiplying the base and the height.

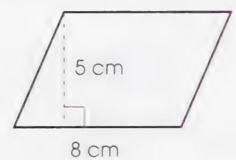
This rule can be expressed by the following formula, where A is the area in square units, b is the base length in linear units, and h is the height in linear units.

$$A = bh$$

You can use this formula to find the area of any parallelogram.

Example

Find the area of this parallelogram.



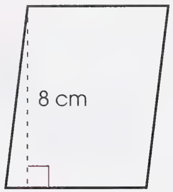
Solution

$$\begin{aligned} A &= bh \\ &= 8(5) \\ &= 40 \end{aligned}$$

The area of the parallelogram is 40 cm^2.

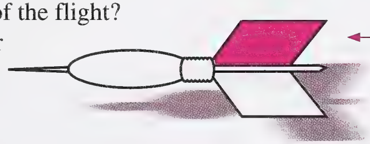
8. Calculate the area of each parallelogram.

a.

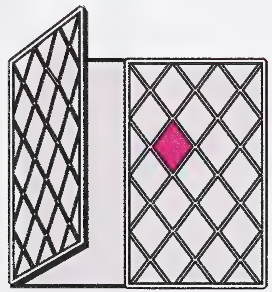


b.

9. The highlighted flight on this dart is in the shape of a parallelogram. The flight's base is 3.2 cm. Its height is 1.7 cm. What is the area of the flight? Round the answer to the nearest tenth of a square centimetre.

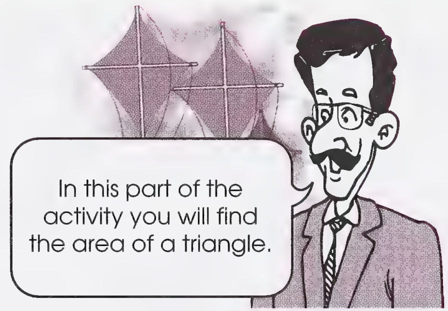


10. The highlighted pane in this window is in the shape of a rhombus. The base of the pane is 14.0 cm. The height of the pane is 12.3 cm. What is the area of the pane of glass?



Check your answers by turning to the Appendix.

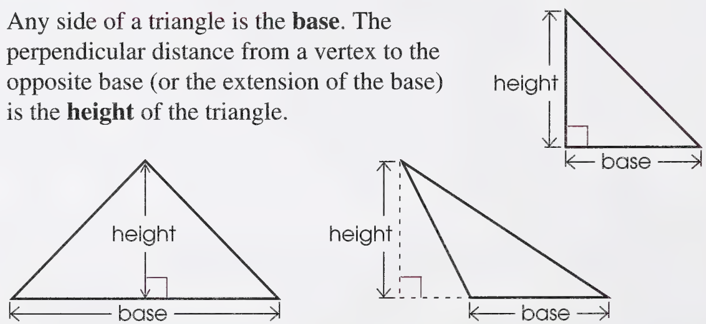
Area of a Triangle



Before beginning your investigation of the area of a triangle, you need to know a few terms.

A triangle with angles each less than 90° is an **acute triangle** (or an **acute-angled triangle**). A triangle with an angle of 90° is a **right triangle** (or a **right-angled triangle**). A triangle with an angle greater than 90° is an **obtuse triangle** (or an **obtuse-angled triangle**).

Any side of a triangle is the **base**. The perpendicular distance from a vertex to the opposite base (or the extension of the base) is the **height** of the triangle.



11. a. Find the page in the Appendix with two congruent triangles drawn on 1-cm grid paper. Give the measurements of the base and height of each triangle.

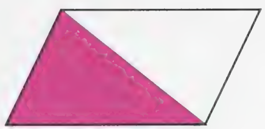
b. Make a photocopy of the page. Cut out the triangles and tape them together to form a parallelogram. Give the measurements of the base and height of the new parallelogram. How do they compare to the base and height of each triangle?

c. Determine the area of the parallelogram.

d. What can you conclude about the area of each triangle?

Check your answers by turning to the Appendix.

You have discovered that a triangle is one-half the area of a parallelogram with the same base and height.



Because of this, the area of the triangle can be found by multiplying the base by the height and then dividing by 2.

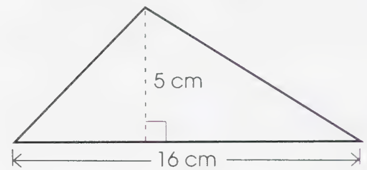
This rule can be expressed by the following formula, where A is the area in square units, b is the base length in linear units, and h is the height in linear units.

$$A = \frac{bh}{2}$$

You can use this formula to find the area of any kind of triangle.

Example 1

Find the area of this triangle.



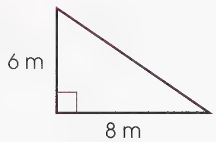
Solution

$$\begin{aligned} A &= \frac{bh}{2} \\ &= \frac{16(5)}{2} \\ &= \frac{80}{2} \\ &= 40 \end{aligned}$$

The area of the triangle is $40\ \text{cm}^2$.

Example 2

Find the area of this triangle.



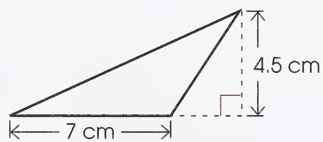
Solution

$$A = \frac{bh}{2}$$
$$= \frac{8(6)}{2}$$
$$= \frac{48}{2}$$
$$= 24$$

The area of the triangle is 24 m^2.

Example 3

Find the area of this triangle.



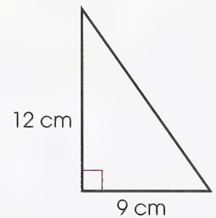
Solution

$$A = \frac{bh}{2}$$
$$= \frac{7(4.5)}{2}$$ ← Use a calculator to multiply and divide.
$$= 15.75$$

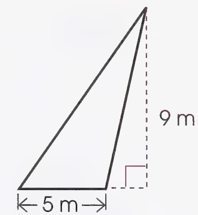
The area of the triangle is 15.75 cm^2.

12. Use the formula to calculate the area of each triangle.

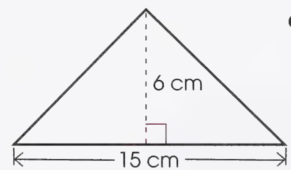
a.



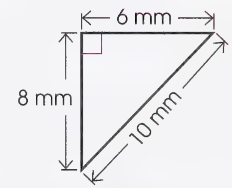
b.



c.



d.



13. This older-style yield sign is triangular in shape. The base is 60 cm and the height is 50 cm. What is the approximate area of the sign? **Note:** The yield signs used today are a slightly different design.

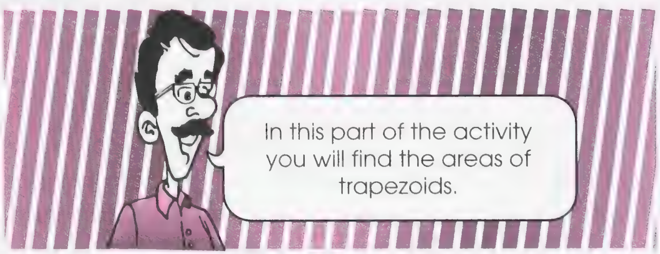
14. Part of the roof on Jacyn's house is shaped like a triangle. The base is 7 m and the height is 3.5 m. What is the area of this part of the roof?

15. a. Aaron wishes to make 18 Titan pennants from a piece of felt that is 1 m × 1 m. Is this possible? **Hint:** Make a sketch using graph paper. **Note:** Felt has no grain (the direction of pieces is not important).

b. What area of felt is used for the 18 pennants?

Check your answers by turning to the Appendix.

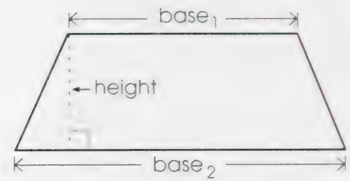
Area of a Trapezoid



Before beginning your investigation of the areas of trapezoids you need to know some terms.

The parallel sides of a trapezoid are called **bases**. The bases are of different lengths, so they are sometimes referred to as $base_1$ and $base_2$.

The perpendicular distance between the bases is the **height**.



16. a. Find the page in the Appendix with two congruent trapezoids drawn on 1-cm grid paper. Give the measurements of the bases and height of each trapezoid.

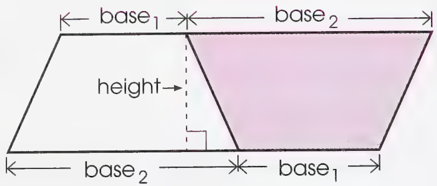
b. Photocopy the page. Cut out the trapezoids and tape them together to form a parallelogram. Give the measurements of the base and height of the new parallelogram. How do these measurements compare to the bases and height of each trapezoid?

c. Determine the area of the parallelogram.

d. What can you conclude about the area of each trapezoid?

Check your answers by turning to the Appendix.

You have discovered that the area of a trapezoid is one-half the area of a parallelogram with the same height and with a base that is the sum of the bases of the trapezoid.



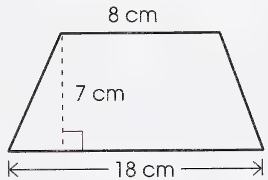
Because of this, the area of any trapezoid can be found by adding base$_1$ and base$_2$, multiplying this sum by the height, and then dividing by 2. This rule can be expressed by the following formula.

$$A = \frac{(b_1 + b_2)h}{2}$$

You can use this formula to find the area of any trapezoid.

Example

Find the area of this trapezoid.



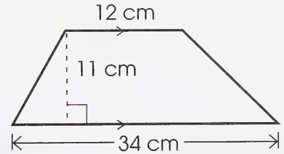
Solution

$$\begin{aligned} A &= \frac{(b_1 + b_2)h}{2} \\ &= \frac{(8+18)7}{2} \\ &= \frac{(26)7}{2} \quad \longleftarrow \text{Use a calculator to multiply and divide.} \\ &= 91 \end{aligned}$$

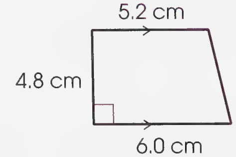
The area of the trapezoid is 91 cm^2.

17. Calculate the area of each of the following trapezoids.

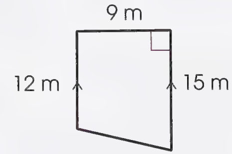
a.



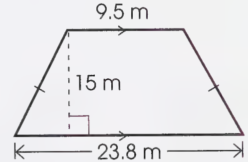
b.



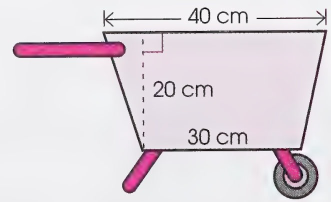
c.



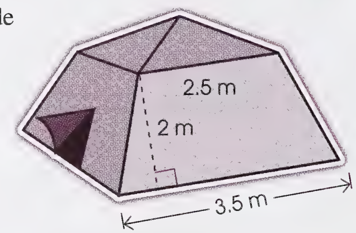
d.



18. Find the area of the given side of a wheel barrow.

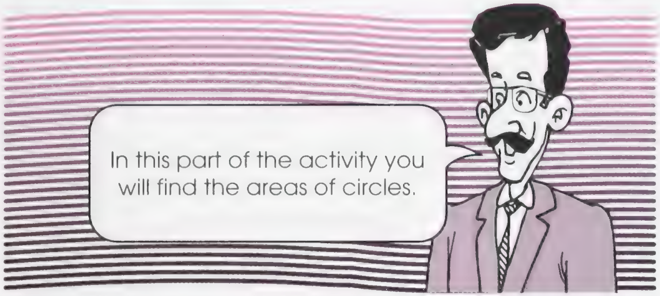


19. Find the area of the given side of the tent.



Check your answers by turning to the Appendix.

Area of a Circle



20. a. In the Appendix, find the page with a diagram of a circle. Measure the diameter and then calculate the circumference of the circle.

b. Colour one-half of the circle. Then cut the circle into pieces. Tape the pieces together to form a shape that approximates a parallelogram, as shown in the following photograph.

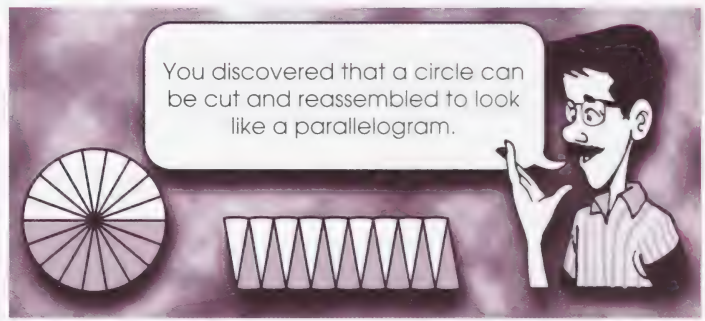
Measure the base of the new "parallelogram." How is the base related to the circle?

Measure the height of the new "parallelogram." How is the height related to the circle?

c. Calculate the area of the new "parallelogram."

d. What can you conclude about the area of the circle?

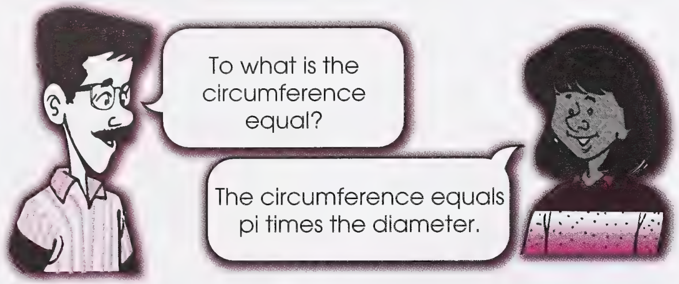
Check your answers by turning to the Appendix.



The base of the "parallelogram" is one-half the circumference of the circle. The height of the "parallelogram" is equal to the radius of the circle.

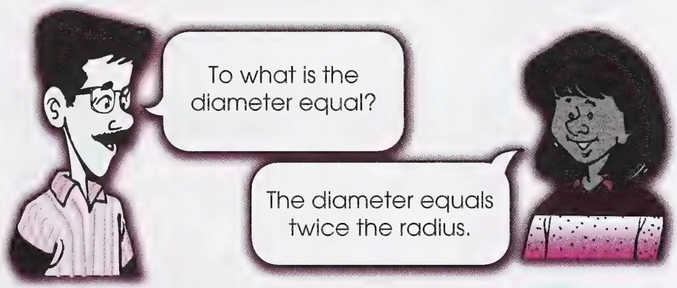
Therefore, the area of the "parallelogram" is equal to the circumference times the radius, divided by 2.

$$\text{Area of "parallelogram"} = \frac{Cr}{2}$$



You can replace the circumference (C) in the equation with πd.

$$\text{Area of "parallelogram"} = \frac{\pi dr}{2}$$



You can replace the diameter (d) in the equation with $2r$ and then simplify.

$$\begin{aligned}\text{Area of "parallelogram"} &= \frac{\pi(2r)r}{2}\\ &= \frac{\pi(\cancel{2}r)r}{\cancel{2}}\\ &= \pi r^2\end{aligned}$$

$r(r) = r^2$

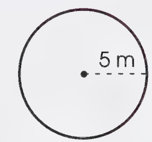
Because the area of the circle equals the area of the "parallelogram," this formula can be used to find the area of a circle:

$$A = \pi r^2$$

You can use this formula to find the area of any circle. Remember, π is about 3.14. **Note:** If your calculator has a key for π, you may use it to perform calculations.

Example

Find the area of this circle, to the nearest square metre.



Solution

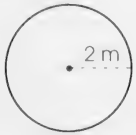
$$\begin{aligned}A &= \pi r^2\\ &\doteq 3.14(5)^2\\ &\doteq 3.14(25)\\ &\doteq 79\end{aligned}$$

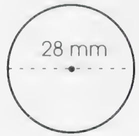
Remember the order of operations.

Use a calculator; you may press the π key instead of entering 3.14.

The area of the circle is about 79 m^2.

21. Calculate the area of each circle. Round each area to the nearest square unit.

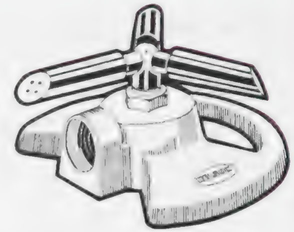
a. 

b. 

22. Munir uses a 10-m rope to tie his horse to a stake in the centre of a field.

If the rope does not become tangled, what is the total area of grass that the horse can eat? Round the area to the nearest square metre.

23. The rotary head on a sprinkler system can spray water a distance of 25 m.



What area of a lawn can be watered by this sprinkler? Round the area to the nearest square metre.

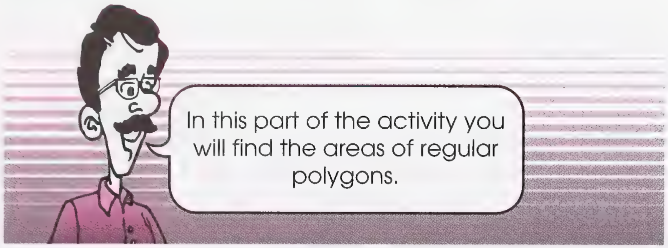
24. How much greater is the area of the face of a quarter than the area of the face of a dime? Round the area to the nearest tenth of a square centimetre. **Hint:** The diameter of a quarter is about 2.4 cm. The diameter of a dime is 1.8 cm.

Check your answers by turning to the Appendix.

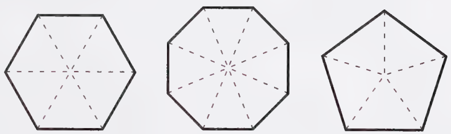
You may wish to explore the Internet to find out more about Canadian coins and the Royal Canadian Mint. This is the uniform resource locator (URL) of a site you may find interesting and entertaining:

http://www.rcmint.ca/

Area of a Regular Polygon



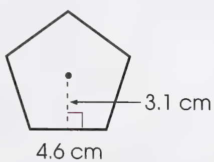
You already know that a regular polygon can be divided into congruent triangles.



This property will help you find the area of a regular polygon.

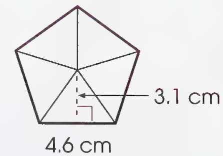
Example 1

Find the area of this regular pentagon.



Solution

Step 1: Divide the pentagon into five congruent triangles and find the area of one triangle.



$$A = \frac{bh}{2}$$
$$= \frac{4.6(3.1)}{2}$$ ← Use a calculator to multiply and divide.
$$= 7.13$$

The area of one triangle is 7.13 cm^2.

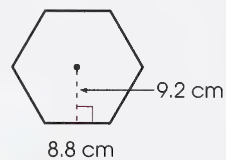
Step 2: Find the area of the entire regular pentagon.

$$5 \times 7.13 = 35.65$$

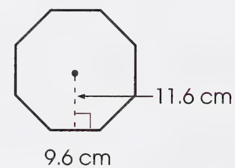
The area of the regular pentagon is 35.65 cm^2.

25. Find the area of each of the following regular polygons.

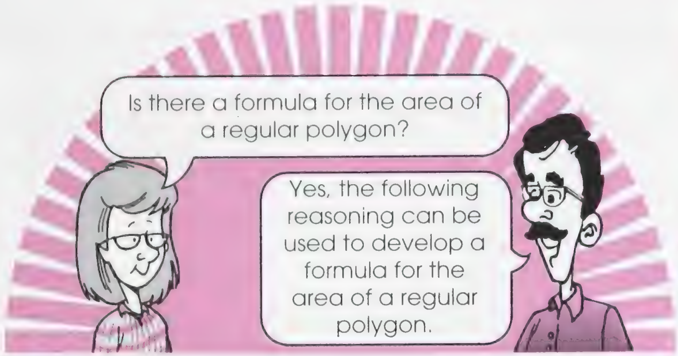
a.



b.



Check your answers by turning to the Appendix.



You already know that the area of a regular polygon is equal to the number of congruent triangles that can be formed in the regular polygon times the area of one of the triangles.

You also know that the number of congruent triangles that can be formed in any regular polygon is equal to the number of sides in the regular polygon.

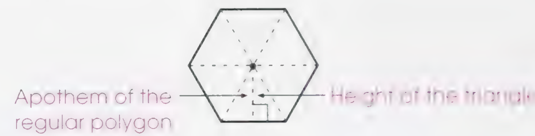
Therefore, the area of a regular polygon is equal to the number of sides in the regular polygon times the area of one triangle.

This rule can be expressed by the following formula, where A is the area of a regular polygon, n is the number of sides in the regular polygon, b is the length of the base of each congruent triangle, and h is the height of each congruent triangle.

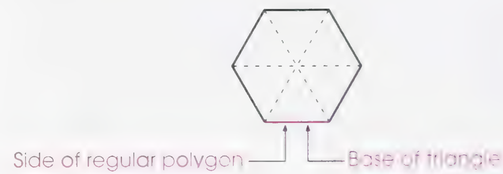
$$A = n\left(\frac{bh}{2}\right)$$
$$= \frac{nbh}{2}$$

The perpendicular distance from the centre of a regular polygon to a side is called the **apothem**.

The apothem of a regular polygon and the height of each congruent triangle formed are the same segment.



Also, the base of the triangle is the same segment as the side of the regular polygon.



Therefore, the area of a regular polygon is equal to the number of sides in the regular polygon times the length of one side times the length of the apothem, divided by two.

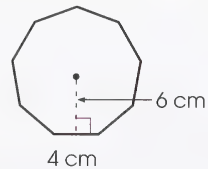
This rule can be expressed by the following formula, where A is the area of a regular polygon, n is the number of sides, s is the length of each side, and a is the length of the apothem.

$$A = \frac{nsa}{2}$$

This example shows how this formula can be used to find the area of a regular polygon.

Example 2

Find the area of this regular nonagon.



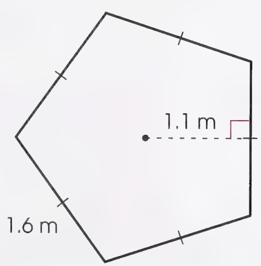
Solution

$$
\begin{aligned}
A &= \frac{nsa}{2} \\
&= \frac{9(4)(6)}{2} \\
&= \frac{216}{2} \\
&= 108
\end{aligned}
$$

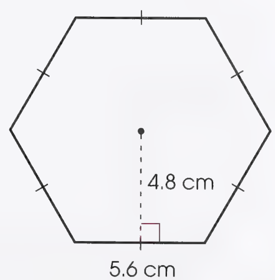
The area of the regular nonagon is 108 cm^2.

26. Calculate the area of each of the following polygons.

a.



b.

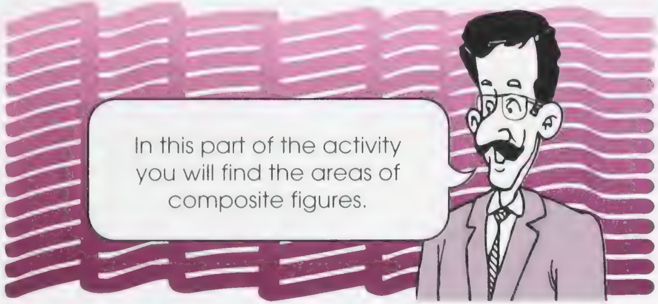


27. A stop sign is a regular octagon, with an apothem of 20 cm and a base of 16.5 cm. What is the area of the sign?

28. The face of a Canadian \$1 coin is a regular hendecagon. Each side of the regular hendecagon is about 7 mm. The apothem is about 13 mm. Calculate the area of a face of the coin.

Check your answers by turning to the Appendix.

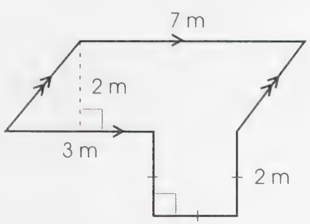
Area of a Composite Figure



The area of a composite figure can be calculated by breaking it into familiar shapes and finding the area of each shape.

Example 1

Find the area of this composite figure.

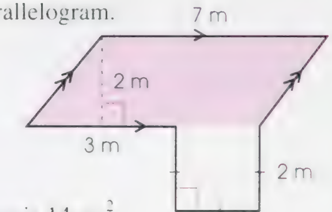


Solution

Step 1: Determine what familiar shapes make up the composite figure.

The composite figure is made up of a parallelogram and a square.

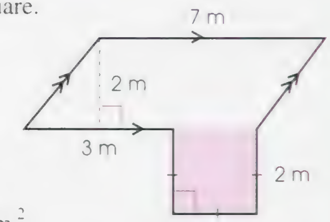
Step 2: Calculate the area of the parallelogram.



$$\begin{aligned} A &= bh \\ &= 7(2) \\ &= 14 \end{aligned}$$

The area of the parallelogram is 14 m^2.

Step 3: Calculate the area of the square.



$$\begin{aligned} A &= s^2 \\ &= 2^2 \\ &= 4 \end{aligned}$$

The area of the square is 4 m^2.

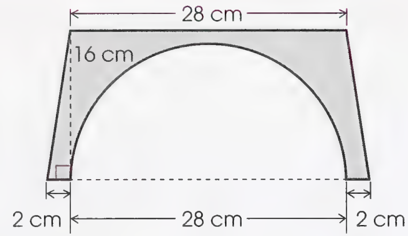
Step 4: Calculate the area of the composite figure.

$$14 + 4 = 28$$

The area of the composite figure is 28 m^2.

Example 2

Find the area of this composite figure. Round the answer to the nearest square centimetre. **Note:** The diagram is not drawn to scale.



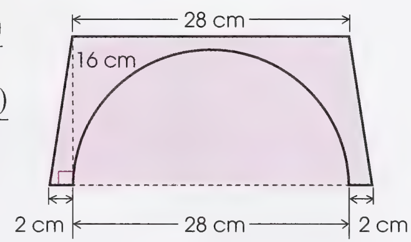
Solution

Step 1: Determine what familiar shapes make up the composite figure.

The composite figure is made up of a trapezoid with a semicircle removed.

Step 2: Find the area of the trapezoid.

$$
\begin{aligned}
A &= \frac{h(b_1+b_2)}{2}\\
&= \frac{16(28+32)}{2}\\
&= \frac{16(60)}{2}\\
&= \frac{960}{2}\\
&= 480
\end{aligned}
$$

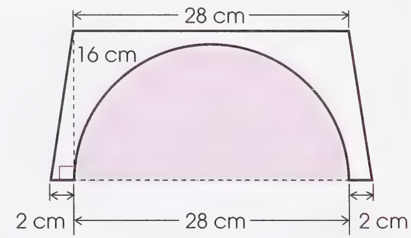


$b_2 = 2+28+2$
$= 32$

The area of the trapezoid is 480 cm^2.

Step 3: Find the area of the semicircle. **Hint:** The area of a semicircle is half the area of a circle with the same diameter.

$$
\begin{aligned}
A &= \pi r^2\\
&\doteq 3.14(14^2)\\
&\doteq 3.14(196)\\
&\doteq 615.44
\end{aligned}
$$



$$
\begin{aligned}
\frac{1}{2}\times 615.44 &\doteq 307.72\\
&\doteq 308
\end{aligned}
$$

$d = 28$
$r = 14$

The area of the semicircle is about 308 cm^2.

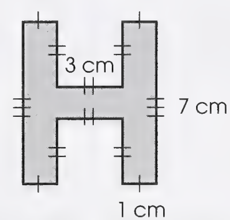
Step 4: Calculate the area of the composite figure.

$480 - 308 = 172$

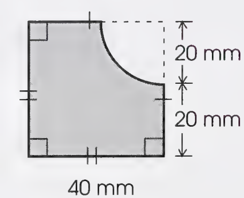
The area of the composite figure is about 172 cm^2.

29. Calculate the area of each of the following shaded figures.

a.

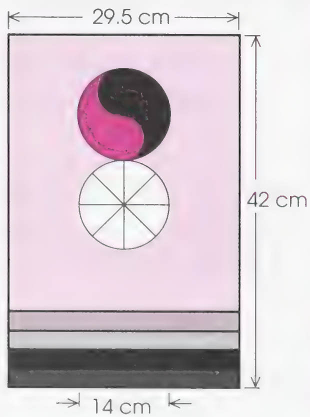


b.

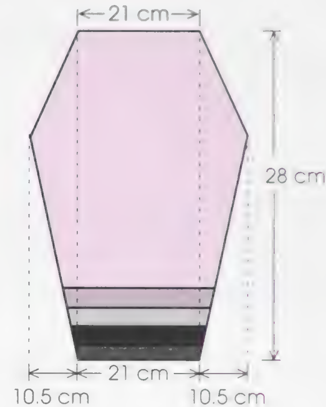


30. Find the area of the outside ring on a Canadian $2 coin. Round the area to the nearest tenth. **Hint:** The diameter of the inside circle is 1.6 cm. The diameter of the coin is 2.8 cm.

31. a. There is a circular hole in the centre of the sail of this Korean Fighter kite. Calculate the area of the sail. **Note:** The diagram is not drawn to scale.



b. Calculate the area of the sail of this Sled kite. **Note:** The diagram is not drawn to scale.



Check your answers by turning to the Appendix.

Did You Know?

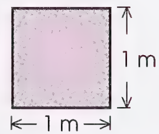
A flat (two-dimensional) kite must have a tail. A tail maintains balance and keeps the kite pointed toward the sky. The more wind there is, the longer the tail should be. A kite should begin with a kite tail at least seven times the diagonal length of the kite's sail.

Use the Internet to find out more about kites and different kite plans.

Now Try This

Use a problem-solving strategy to answer the following questions.

32. How can you cover half of a 1-m square window and still have a square opening that is 1 m across and 1 m from top to bottom?

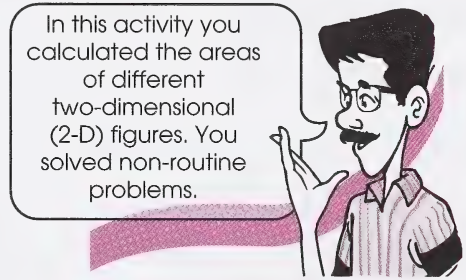


33. A goat is tied by a rope to a stake at the corner of a barn. The barn's dimensions are as shown in the following diagram.

a. If the rope is 8 m long, over what area can the goat graze? Round your answer to the nearest square metre.

b. If the rope is 12 m long, over what area can the goat graze? Round your answer to the nearest square metre.

Check your answers by turning to the Appendix.

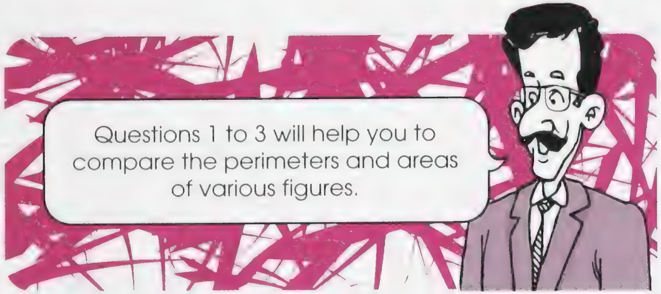


Follow-up Activities

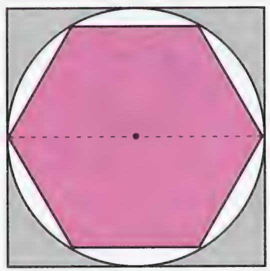
If you had difficulties understanding the concepts and skills in the activities, it is recommended that you do the Extra Help. If you have a clear understanding of the concepts and skills, it is recommended that you do the Enrichment. You may decide to do both.

Extra Help

In this section you calculated the perimeters of various polygons and the circumferences of circles.



1. In the given diagram the diameter of the circle is 2 cm, each side of the square is 2 cm, and each side of the regular hexagon is 1 cm.



   a. Calculate the perimeter of the square using the formula $P = ns$.

   b. Calculate the perimeter of the hexagon using the formula $P = ns$.

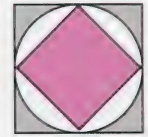
   c. What can you conclude about the circumference of the circle?

   d. Calculate the circumference of the circle using the formula $C = \pi d$.

Check your answers by turning to the Appendix.

In this section you calculated the areas of various figures.

2. In the given diagram the diameter of the circle is 2 cm, each side of the larger square is 2 cm, and each side of the smaller square is 1.4 cm.



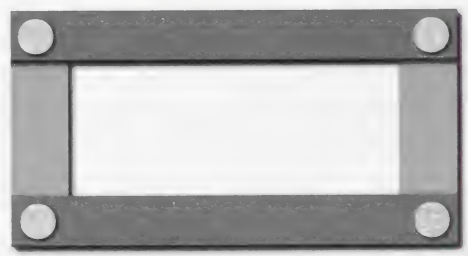
   a. Calculate the area of the larger square using the formula $A = s^2$.

   b. Calculate the area of the smaller square using the formula $A = s^2$.

   c. What can you conclude about the area of the circle?

   d. Calculate the area of the circle by using the formula $A = \pi r^2$.

3. Cut out two pairs of rectangular strips of stiff paper (card stock); the pairs may be different lengths. As shown in the following diagram, fasten the strips together with paper fasteners to form a rectangle.



Take hold of two opposite corners and pull. Keep the opposite angles equal.

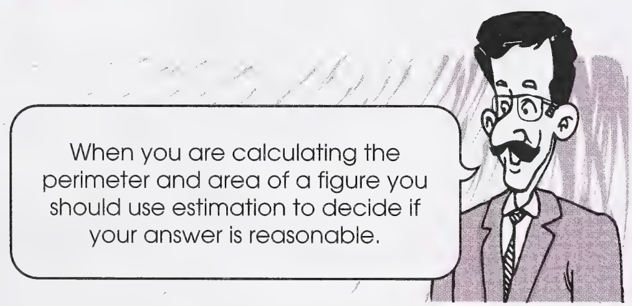
a. As you pull, what new shape is formed?

b. As you pull, does the base of the new shape increase, decrease, or remain the same?

c. As you pull, does the height of the new shape increase, decrease, or remain the same?

d. What can you conclude about the area of the new shape? As you pull, does the area increase, decrease, or remain the same? Explain.

Check your answers by turning to the Appendix.



Example 1

[1]

Estimate the area of the picture on this Canadian stamp. The length of the picture is 2.1 cm and the width is 1.6 cm.

Solution

Step 1: Estimate the area. Use the formula for finding the area of a rectangle.

Rounding

$$A = lw$$
$$\doteq 2 \times 2$$
$$\doteq 4$$

The area is about $4\ \text{cm}^2$.

Front-end Digits

$$A = lw$$
$$\doteq 2 \times 1$$
$$\doteq 2$$

The area is about $2\ \text{cm}^2$.

Step 2: Calculate the area.

$$A = lw$$
$$= 2.1 \times 1.6 \quad \longleftarrow \text{Use a calculator to multiply.}$$
$$= 3.36$$

The area of the picture on this stamp is $3.36\ \text{cm}^2$.

[1] Courtesy of Canada Post Corporation

Step 3: Compare the calculated answer and the estimate to decide if your answer is reasonable.

$3.36 \doteq 4$ $\qquad$ $3.36 \doteq 2$

Therefore, the answer is reasonable.

The area of the stamp is $3.36\ \text{cm}^2$.

4. Calculate the area of the picture on each of the following international stamps. Be sure to estimate to decide if your answer is reasonable.

a.

[1]

Base: 5.0 cm $\qquad$ **Height:** 2.5 cm

b.

[2]

Base: 3.4 cm $\qquad$ **Height:** 2.9 cm

[1] Courtesy of Azienda Autonoma Di Stato Filatelica E Numismatica, Repubblica Di San Marino
[2] Courtesy of Division de Especies Postales y Falatelia, Nicaragua

c.

[1]

Base: 3.5 cm $\qquad$ **Height:** 3.5 cm

d.

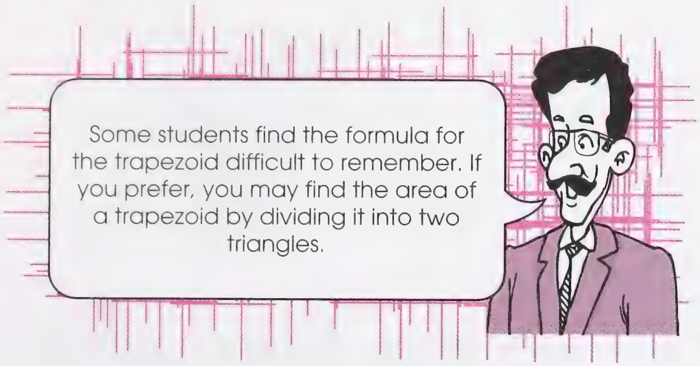
[2]

Base: 3.1 cm $\qquad$ **Height:** 2.7 cm

Check your answers by turning to the Appendix.

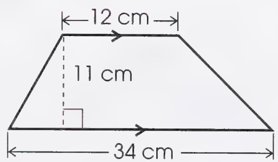
[1] Courtesy of Office des Emissions de Timbres-Poste, Principaute de Monaco
[2] Courtesy of Pos Malaysia

You may wish to use the Internet to explore stamp collecting. Use your search engines to explore the terms *stamp collecting* or *philately*.



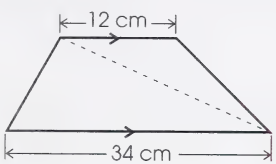
Example 2

Calculate the area of this trapezoid.



Solution

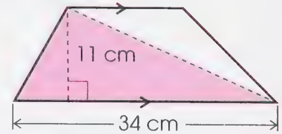
Step 1: Divide the trapezoid into two triangles.



Each triangle has a height of 11 cm.

Step 2: Find the area of one triangle.

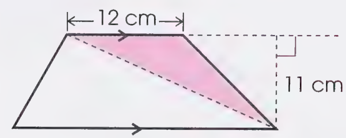
$$\begin{aligned} A &= \frac{bh}{2} \\ &= \frac{34(11)}{2} \\ &= \frac{374}{2} \\ &= 187 \end{aligned}$$



The area is 187 cm^2.

Step 3: Find the area of the other triangle. **Hint:** The two triangles have the same height because the bases of the trapezoid are parallel.

$$\begin{aligned} A &= \frac{bh}{2} \\ &= \frac{12(11)}{2} \\ &= \frac{132}{2} \\ &= 66 \end{aligned}$$



The area is 66 cm^2.

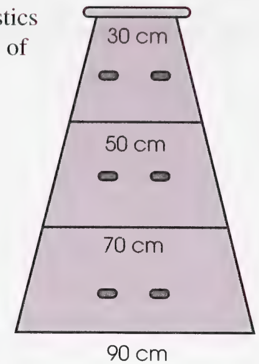
Step 4: Find the area of the trapezoid.

$$187 + 66 = 253$$

The area of the trapezoid is 253 cm^2.

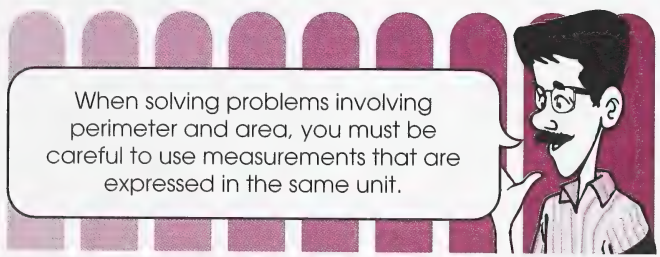
5. The side of a vaulting horse for gymnastics is shaped like a trapezoid. It is made up of three sections that lock together. Each section is 40 cm high.

Find the area of the given side of each section.



Check your answers by turning to the Appendix.

Enrichment



It is easy to change from one unit to another in the metric system because the units are multiples of ten.

1 km = 10 hm	1 hm = 10 dam	1 dam = 10 m
1 m = 10 dm	1 dm = 10 cm	1 cm = 10 mm

You can use the following metric ladder to help you remember metric units of length.

larger units ← → smaller units

km	hm	dam	m	dm	cm	mm

Example 1

Change 20 cm to metres.

Solution

Use the metric ladder.

fewer larger units ← → more smaller units

km	hm	dam	m	dm	cm	mm

$\div(10 \times 10)$

To change from centimetres to metres, divide by 100. (Dividing by 100 moves the decimal point two places to the left.)

$$20 \text{ cm} = 0.2 \text{ m}$$

Note: Because you are changing to larger units, there will be fewer metres than centimetres.

Example 2

Change 30 cm to millimetres.

Solution

Use the metric ladder.

fewer larger units — more smaller units

km	hm	dam	m	dm	cm	mm

$\times 10$ (cm → mm)

To change from centimetres to millimetres, multiply by 10. (Multiplying by 10 moves the decimal point one place to the right.)

$$30 \text{ cm} = 300 \text{ mm}$$

Note: Because you are changing to smaller units, there will be more millimetres than centimetres.

1. **a.** How do you change from kilometres to metres?
   b. How do you change from millimetres to metres?

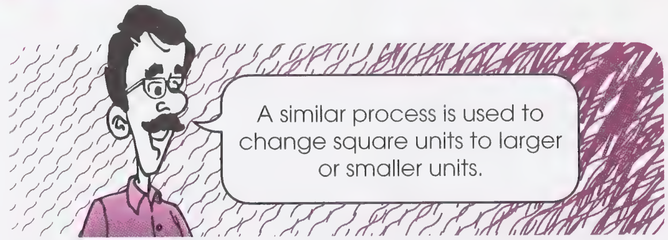
2. Change each of the following measurements to centimetres.

   a. 5 m **b.** 80 m **c.** 65 mm
   d. 0.7 m **e.** 45 mm **f.** 173 mm

3. Change each of the following measurements to metres.

   a. 200 cm **b.** 3 km **c.** 8000 mm
   d. 53.8 cm **e.** 5.9 km **f.** 732 mm

Check your answers by turning to the Appendix.



It is easy to change from one square unit to another in the metric system because the units are multiples of 100.

$1 \text{ km}^2 = 100 \text{ hm}^2$ $1 \text{ hm}^2 = 100 \text{ dam}^2$ $1 \text{ dam}^2 = 100 \text{ m}^2$
$1 \text{ m}^2 = 100 \text{ dm}^2$ $1 \text{ dm}^2 = 100 \text{ cm}^2$ $1 \text{ cm}^2 = 100 \text{ mm}^2$

You can use the metric ladder to help you remember the metric units of area.

larger units — smaller units

km^2	hm^2	dam^2	m^2	dm^2	cm^2	mm^2

Example 3

Change 3.8 m^2 to square centimetres.

Solution

Use the metric ladder.

fewer larger units ← → more smaller units

km^2	hm^2	dam^2	m^2	dm^2	cm^2	mm^2

$\times(100\times100)$

To change from square metres to square centimetres, multiply by 10 000.

$$3.8\ m^2 = 38\ 000\ cm^2$$

Note: Because you are changing to smaller units, there will be more square centimetres than square metres.

4. **a.** How do you change from square kilometres to square metres?

b. How do you change from square millimetres to square metres?

5. Change each of the following measurements to square centimetres.

a. 250 m^2 **b.** 0.5 m^2 **c.** 60 mm^2

d. 850 mm^2 **e.** 3 m^2 **f.** 9000 mm^2

6. Change each of the following measurements to square metres.

a. 80 000 cm^2 **b.** 6000 cm^2 **c.** 3 km^2

d. 8000 mm^2 **e.** 50 km^2 **f.** 550 mm^2

Check your answers by turning to the Appendix.

You are now ready to solve problems involving changes of units.

7. A square room has sides of 9 m. Its floor is to be covered with square tiles having sides of 30 cm. How many tiles will be needed?

8. Verna shovelled a walk that is 80 cm wide and 5 m long. What is the area that Verna shovelled?

9. Highways vary in length and width.

The Trans-Canada highway is about 7820 km long. Its average width is 7.2 m. Calculate the approximate area of the highway. Round your answer to the nearest square kilometre.

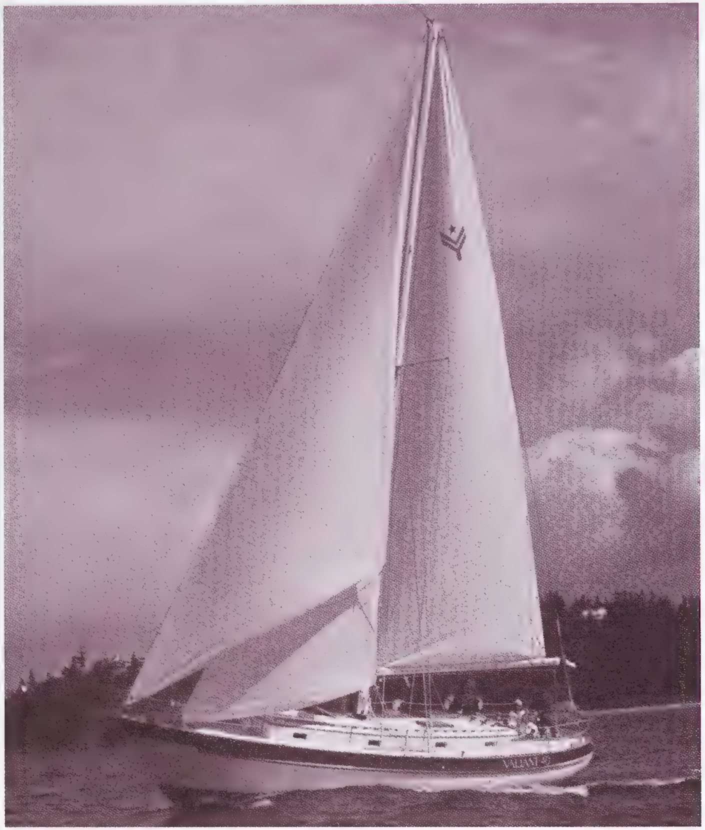
10. A rectangular field, which measures 5 km $\times$ 4 km, is inhabited by a colony of prairie dogs.

a. What area of land is inhabited by the colony?

b. If there are 40 000 000 prairie dogs in the colony, how many prairie dogs are there per square metre?

Check your answers by turning to the Appendix.

Conclusion



In this section you generalized measurement patterns and procedures and solved problems involving perimeter and area.

Area is an important factor to consider in many situations. For example, area is a factor when walking in deep snow. If you are wearing snowshoes you will sink less and expend less energy because your mass is distributed over a greater area.

Do you think that the area of a sailboat's sails helps the boat skim across the water? How will the sailboat's speed be affected if one of the sails is lowered?

Assignment

Assignment Booklet

You are now ready to complete the assignment for Section 2.

Module Summary

In this module you explored two-dimensional geometry. You discovered the properties of plane (flat) figures. You used these properties to solve problems. You generalized measurement patterns and procedures and solved problems involving perimeter and area.

Two-dimensional geometry is used in designing many objects. For example, kite builders must take many factors into consideration when they make a kite.

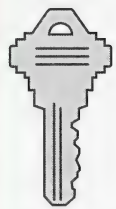
Why do you think a kite sail is symmetric? Does the area of a kite's sail affect the way it flies?

Final Module Assignment

Assignment Booklet

You are now ready to complete the final module assignment.

APPENDIX



- Glossary
- Suggested Answers
- Articles/Puzzles
- Cut-out Learning Aids

Glossary

Acute triangle: a triangle with each angle less than 90°; also called an **acute-angled triangle**

Adjacent angles: pairs of angles sharing a common side and a common vertex

Apothem: the perpendicular distance from the centre of a regular polygon to a side

Arc: the part of a circle between any two points on the circle

Base (of a parallelogram): any side of a parallelogram

Bases (of a trapezoid): the parallel sides of a trapezoid; sometimes called $base_1$ and $base_2$ because they are of different lengths

Central angle: the angle formed by a pair of radii

Chord: a line segment joining two points on a circle

Circle: a set of all points in a plane that are the same distance from a fixed point called the **centre**

Circumference: the distance around a circle

Composite (figure): a figure made up of two or more shapes

Dart: a concave trapezium with two sets of adjacent sides congruent; also called an **arrowhead** or a **deltoid**

Degree (of a vertex): the number of edges that meet the vertex

Diagonal: a line segment joining any two vertices of a polygon not already joined

Diameter: a chord through the centre of a circle

Even vertex (of a network): a vertex having an even number of edges connected to it

Height (of a parallelogram): in a parallelogram, the perpendicular distance from the base to the opposite side

Height (of a trapezoid): the perpendicular distance between the bases of a trapezoid

Inscribed: a figure drawn inside a circle or other figure, with all the points of the inner figure touching the outer figure

Inscribed angle: an angle drawn inside a circle so that the vertex of the angle is on the circle and the arms of the angle are chords of the circle

Isosceles trapezoid: a trapezoid with the non-parallel sides congruent

Kite: a convex trapezium with two sets of adjacent sides congruent

Network: a figure consisting of edges and vertices; sometimes called a **graph**

Obtuse triangle: a triangle with an angle greater than 90°; also called an **obtuse-angled triangle**

Odd vertex (of a network): a vertex having an odd number of edges connected to it

Opposite angles (in a parallelogram): non-adjacent angles

Parallelogram: a quadrilateral with two pairs of parallel sides

Perimeter: the distance around a figure

Polygon: a simple closed figure with straight sides

Problem: a task for which the method of finding the answer (as well as the answer) is not immediately known

Radius: a line segment from the centre of a circle to any point on the circle

Rectangle: a parallelogram with a right angle

Regular polygon: a polygon with congruent sides and congruent angles

Rhombus: a parallelogram with four congruent sides

Right triangle: a triangle with an angle of 90°; also called a **right-angled triangle**

Sector: the region in a circle bounded by a pair of radii and an arc

Square: a parallelogram with four congruent sides and a right angle

Supplementary (angles): two angles having a sum of 180°

Symmetry: the property that makes a figure look balanced

Tangram: an ancient Chinese puzzle that has seven geometric shapes called **tans** (two large triangles, one medium triangle, two small triangles, a square, and a parallelogram)

Technology: the application of tools, materials, and processes to the solution of problems; more specifically, devices and systems used in processing, transferring, storing, and communicating information through electronic media

Tessellation: an arrangement of congruent figures that covers a surface without gaps or overlapping

Traversable: a network that has a path that travels along every edge exactly once

Trapezium: a quadrilateral without any parallel sides

Trapezoid: a quadrilateral with exactly one pair of parallel sides

Suggested Answers

Section 1: Activity 1

1. **a.** Yes

b. No, the sides are not all straight.

c. No, the figure is not simple; there are crossovers.

d. No, the figure is not closed; it does not have an inside and an outside.

e. Yes   f. Yes

2. a. pentagon   b. nonagon   c. dodecagon
   d. decagon   e. triangle   f. quadrilateral
   g. heptagon   h. octagon

3. a. hendecagon   b. octagon
   c. heptagon   d. quadrilateral
   e. hexagon   f. pentagon; quadrilateral

4. a.

Name of Polygon	Number of Sides	Number of Triangles	Sum of the Interior Angles
Triangle	3	1	180°
Quadrilateral	4	2	360°
Pentagon	5	3	540°
Hexagon	6	4	720°
Heptagon	7	5	900°
Octagon	8	6	1080°

b. $t = n - 2$ c. $s = (n-2)180°$

5. a. **Step 1:** The figure is a pentagon; therefore, calculate the sum of the interior angles of a pentagon.

$$\begin{aligned} s &= (n-2)180° \\ &= (5-2)180° \\ &= (3)180° \\ &= 540° \end{aligned}$$

Step 2: Find the measure of the unknown angle.

$$\begin{aligned} b + 80° + 140° + 120° + 110° &= 540° \\ b + 450° &= 540° \\ b &= 90° \end{aligned}$$

The unknown angle is 90°.

b. **Step 1:** The figure is a hexagon; therefore, calculate the sum of the interior angles of a hexagon.

$$\begin{aligned} s &= (n-2)180° \\ &= (6-2)180° \\ &= (4)180° \\ &= 720° \end{aligned}$$

Step 2: Find the measure of the unknown angle.

$$\begin{aligned} a + 150° + 130° + 90° + 135° + 95° &= 720° \\ a + 600° &= 720° \\ a &= 120° \end{aligned}$$

The unknown angle is 120°.

c. Step 1: The figure is a quadrilateral; therefore, calculate the sum of the interior angles of a quadrilateral.

$$\begin{aligned} s &= (n-2)180^\circ \\ &= (4-2)180^\circ \\ &= (2)180^\circ \\ &= 360^\circ \end{aligned}$$

Step 2: Find the measure of the unknown angle.

$$\begin{aligned} c + 120^\circ + 85^\circ + 95^\circ &= 360^\circ \\ c + 300^\circ &= 360^\circ \\ c &= 60^\circ \end{aligned}$$

The unknown angle is 60°.

d. Step 1: The figure is a heptagon; therefore, calculate the sum of the interior angles of a heptagon.

$$\begin{aligned} s &= (n-2)180^\circ \\ &= (7-2)180^\circ \\ &= (5)180^\circ \\ &= 900^\circ \end{aligned}$$

Step 2: Find the measure of the unknown angle.

$$\begin{aligned} d + 35^\circ + 90^\circ + 250^\circ + 95^\circ + 70^\circ + 120^\circ &= 900^\circ \\ d + 660^\circ &= 900^\circ \\ d &= 240^\circ \end{aligned}$$

The unknown angle is 240°.

6.

Name of Polygon	Number of Sides	Sum of the Interior Angles	Measure of Each Interior Angle
Regular Triangle	3	180°	60°
Regular Quadrilateral	4	360°	90°
Regular Pentagon	5	540°	108°
Regular Hexagon	6	720°	120°
Regular Heptagon	7	900°	129° (approximately)
Regular Octagon	8	1080°	135°

7. a.

Name of Polygon	Number of Lines of Symmetry	Order of Turn Symmetry
Regular Triangle	3	3
Regular Quadrilateral	4	4
Regular Pentagon	5	5
Regular Hexagon	6	6
Regular Heptagon	7	7
Regular Octagon	8	8

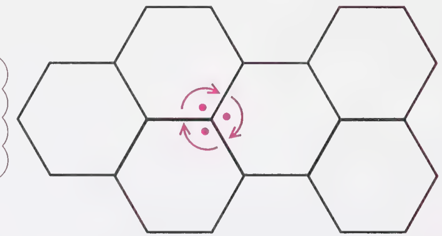
b. All the regular polygons have symmetry. The number of lines of symmetry equals the number of sides. The order of turn symmetry equals the number of sides.

c. Yes, a regular decagon has flip symmetry. It has 10 lines of symmetry.

d. Yes, a regular decagon has turn symmetry. The order of turn symmetry is 10.

8. a. Congruent regular hexagons tessellate because the sum of the angles where the regular hexagons meet is 360°, a complete circle.

Each interior angle of a regular hexagon is 120°.
$3 \times 120° = 360°$

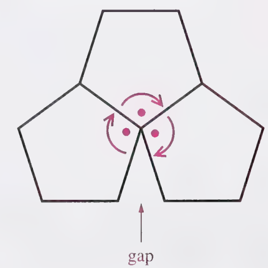


b. Congruent regular pentagons don't tessellate because the sum of the angles where the pentagons meet is not 360°, a complete circle.

Each interior angle of a regular pentagon is 108°.

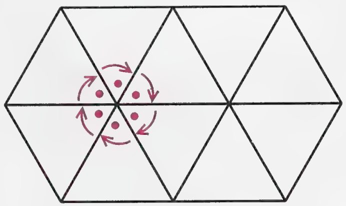
$3 \times 108° = 324°;\ 324° < 360°$

$4 \times 108° = 432°;\ 432° > 360°$



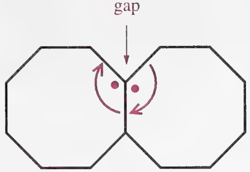
There will be a gap with three regular pentagons; however, the gap is not large enough for a fourth regular pentagon.

9. **a.** Yes, congruent regular triangles tessellate. The reason is that each angle in a regular triangle is 60°, and 60° divides evenly into 360°.



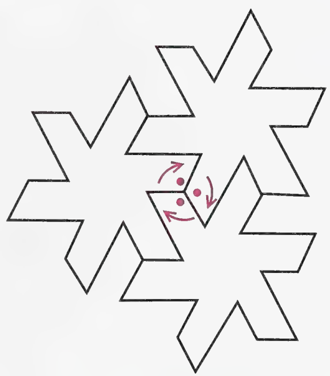
$360° \div 60° = 6$

b. No, congruent regular octagons do not tessellate. The reason is that each angle in a regular octagon is 135°, and 135° does not divide evenly into 360°.



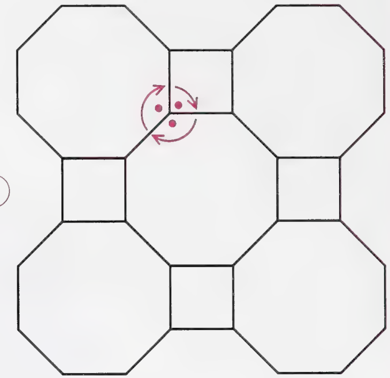
$360° \div 135° \doteq 2.7$

10. The shape tessellates because all the corresponding sides are of equal length, and the sum of the angles where the shapes meet is 360°.

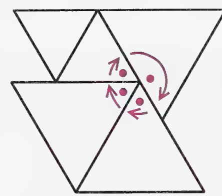


11. This regular octagon and regular quadrilateral tessellate because all the corresponding sides are of equal length, and the sum of the angles where the figures meet is 360°.

$90° + 135° + 135° = 360°$

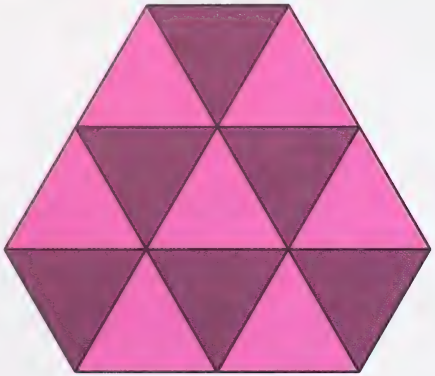


12. These two different sizes of regular triangles tessellate because the sum of the angles where the figures meet is 360°.

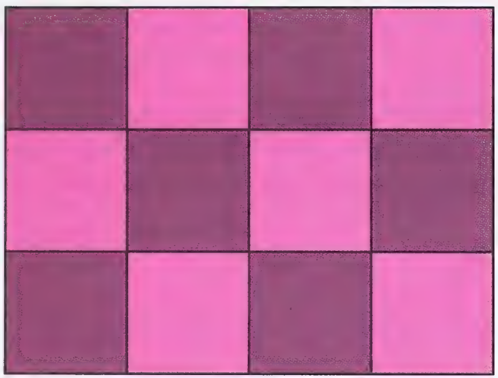


$180° + 60° + 60° + 60° = 360°$

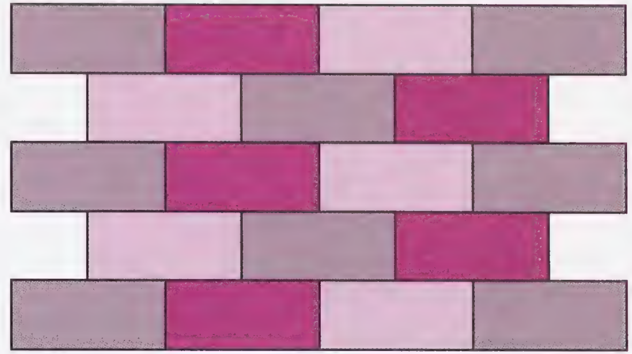
13. **a.** The minimum number of colours required is 2.



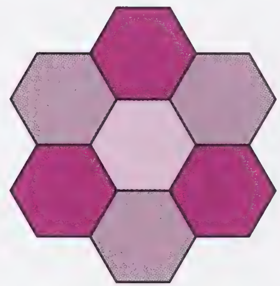
b. The minimum number of colours required is 2.



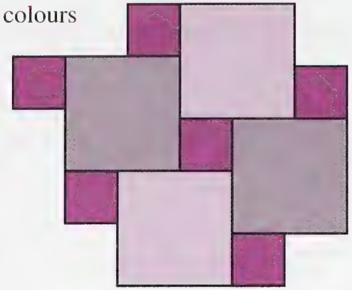
c. The minimum number of colours required is 3.



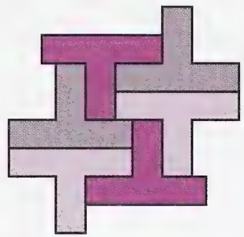
d. The minimum number of colours required is 3.



e. The minimum number of colours required is 3.



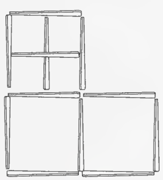
f. The minimum number of colours required is 3.



Now Try This

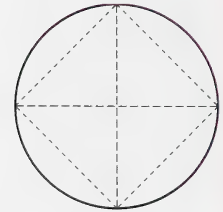
14. You may have to change your point of view to answer this question.

There are a total of seven squares in the following figure. Other answers are possible.

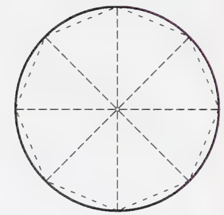


Section 1: Activity 2

1. When the circle is unfolded, a regular quadrilateral (square) is produced. The figure has 4 congruent sides and 4 congruent angles.



2. When the circle is unfolded, a regular octagon is produced. The figure has 8 congruent sides and 8 congruent angles.



3. **a.** $360° \div 5 = 72°$

Each of the central angles is 72°.

b. $360° \div 6 = 60°$

Each of the central angles is 60°.

c. $360° \div 9 = 40°$

Each of the central angles is 40°.

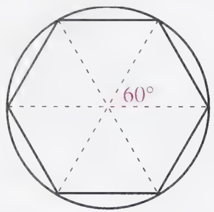
d. $360° \div 10 = 36°$

Each of the central angles is 36°.

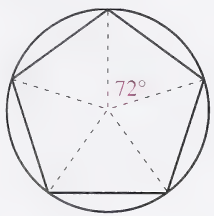
4. a. regular hendecagon
   b. regular pentagon
   c. regular heptagon
   d. regular decagon

5. The number of central angles drawn is equal to the number of sides in the inscribed regular polygon.

6. a. Your regular hexagon should look like this.



   b. Your regular pentagon should look like this.



7. Drawings will vary. Two or more inscribed angles subtended by the same arc are equal.

8. Drawings will vary. If an inscribed angle and a central angle are subtended by the same arc, the measure of the inscribed angle is half the measure of the central angle.

9. a.

Statement	Reason
$a = \frac{1}{2} \times 120°$	An inscribed angle is one-half the measure of a central angle subtended by the same arc.
$\therefore\ a = 60°$	

b.

Statement	Reason
$b = 50°$	Inscribed angles subtended by the same arc are equal.

c.

Statement	Reason
$48° = \frac{1}{2}z$	An inscribed angle is one-half the measure of a central angle subtended by the same arc.
$\therefore\ z = 96°$	

d.

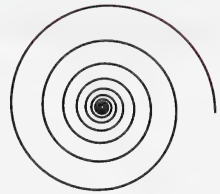
Statement	Reason
$\angle LMN = 180°$	A straight angle has 180°.
$o = \frac{1}{2} \times 180°$	An inscribed angle is one-half of the measure of a central angle subtended by the same arc.
$\therefore\ o = 90°$	

Did You Know?

10. a. Scott Abbott and Chris Haney, two Montreal journalists, invented Trivial Pursuit®.

    b. The game was invented in 1979.

c. There is only one groove on one side of a record. The groove spirals like this.



Now Try This

11. You can solve this problem by using reasoning and working backwards.

Step 1: Use this reasoning. Subtracting $\frac{1}{2}$ of the audience is the same as multiplying the audience by $\frac{1}{2}$. Subtracting $\frac{1}{3}$ of the remaining audience is the same as multiplying the remaining audience by $\frac{2}{3}$. Subtracting $\frac{1}{4}$ of the remaining audience is the same as multiplying the remaining audience by $\frac{3}{4}$.

Step 2: Make a flow chart and a reverse flow chart.

Flow Chart

? → Multiply by $\frac{1}{2}$. → Multiply by $\frac{2}{3}$. → Multiply by $\frac{3}{4}$. → 9

Reverse Flow Chart

? ← Divide by $\frac{1}{2}$. ← Divide by $\frac{2}{3}$. ← Divide by $\frac{3}{4}$. ← 9

Step 3: Use the reverse flow chart to find the number of people in the audience at the beginning.

$$9 \div \frac{3}{4} = 9 \times \frac{4}{3} = 12$$

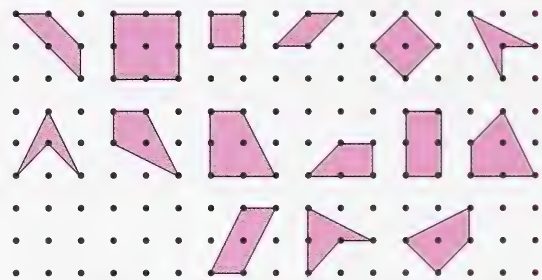
$$12 \div \frac{2}{3} = 12 \times \frac{3}{2} = 18$$

$$18 \div \frac{1}{2} = 18 \times \frac{2}{1} = 36$$

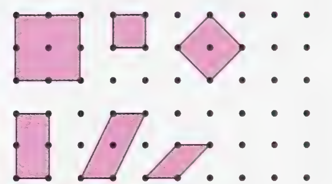
There were 36 people in the audience at the beginning.

Section 1: Activity 3

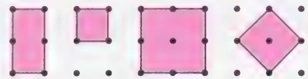
1. The 15 quadrilaterals are shown in the following diagram.



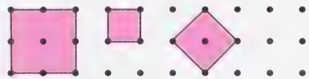
2. These 6 quadrilaterals are parallelograms.



3\. a. These 4 quadrilaterals are rectangles.

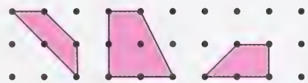


b. Each of these 3 figures is a rhombus.

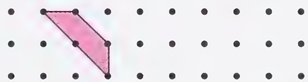


c. Each rhombus in 3.b. is also a square.

4\. a. These 3 quadrilaterals are trapezoids.

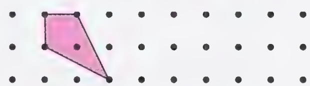


b. This quadrilateral is an isosceles trapezoid.



5\. a. These 6 quadrilaterals are trapeziums.

b. This quadrilateral is a kite.



c. These 2 quadrilaterals are darts.

6\. **a.** yes **b.** yes **c.** no **d.** yes
e. yes **f.** no **g.** no **h.** no
i. no **j.** yes

7\. **a.** isosceles trapezoid **b.** rhombus
c. rectangle **d.** kite
e. parallelogram

8\. **a.** rectangle **b.** isosceles trapezoid
c. parallelogram **d.** rhombus
e. square **f.** dart

9\. **a.** $\angle 1$ and $\angle 3$ are opposite angles. $\angle 2$ and $\angle 4$ are opposite angles.

b. $\angle 1$ and $\angle 2$ are adjacent angles. $\angle 2$ and $\angle 3$ are adjacent angles. $\angle 3$ and $\angle 4$ are adjacent angles. $\angle 1$ and $\angle 4$ are adjacent angles.

10\. **a.** $\angle 1 = \angle 3$ because they are corresponding angles of parallel lines.

b. $\angle 3 = \angle 5$ because they are interior alternate angles of parallel lines.

c. Because $\angle 1 = 3$ and $\angle 3 = \angle 5$, you conclude that in parallelogram *ABCD* the opposite angles $\angle 1$ and $\angle 5$ are equal.

d. $\angle 2 = \angle 6$ because they are corresponding angles of parallel lines.

e. $\angle 6 = \angle 4$ because they are interior alternate angles of parallel lines.

f. Because $\angle 2 = \angle 6$ and $\angle 6 = \angle 4$, you can conclude that in parallelogram *ABCD* the opposite angles $\angle 2$ and $\angle 4$ are equal.

g. Opposite angles in a parallelogram are equal.

11. a. $\angle 2$ and $\angle 3$ are supplementary angles because they form a straight angle.

b. $\angle 1 = \angle 3$ because they are corresponding angles of parallel lines.

c. $\angle 3 = \angle 5$ because they are interior alternate angles of parallel lines.

d. Because $\angle 2$ and $\angle 3$ are supplementary and $\angle 1 = \angle 3$, you can conclude that in parallelogram *ABCD* the adjacent angles $\angle 1$ and $\angle 2$ are supplementary.

e. Because $\angle 2$ and $\angle 3$ are supplementary and $\angle 3 = \angle 5$, you conclude that in parallelogram *ABCD* the adjacent angles $\angle 2$ and $\angle 5$ are supplementary.

f. Adjacent angles in a parallelogram are supplementary.

12. a.

Statement	Reason
$u = 130°$	Opposite angles of a parallelogram are equal.
$r + 130° = 180°$ $\therefore\ r = 50°$	Adjacent angles of a parallelogram are supplementary.
$t = 50°$	Opposite angles of a parallelogram are equal.

$\therefore\ u = 130°,\ r = 50°,$ and $t = 50°$

b.

Statement	Reason
$\angle z = 90°$	given
$x = 90°$	Opposite angles of a parallelogram are equal.
$w + 90° = 180°$ $\therefore\ w = 90°$	Adjacent angles of a parallelogram are supplementary.
$y = 90°$	Opposite angles of a parallelogram are equal.

$\therefore\ x = 90°,\ w = 90°,$ and $y = 90°$

13.

Name of Quadrilateral	Are the Two Diagonals of Equal Length?	Do the Two Diagonals Bisect Each Other?	Do the Two Diagonals Cross at Right Angles?
Parallelogram	no	yes	no
Rhombus	no	yes	yes
Rectangle	yes	yes	no
Square	yes	yes	yes
Trapezoid	no	no	no
Isosceles Trapezoid	yes	no	no
Trapezium	no	no	no
Kite	no	no	yes
Dart	N/A	N/A	N/A

14. a. Rectangles, squares, and isosceles trapezoids have diagonals of equal length.

b. The diagonals of all parallelograms (including rhombuses, rectangles, and squares) bisect each other.

c. The diagonals of rhombuses, squares, and kites meet at right angles.

15. a. In Rectangle *ABCD*, several pairs of line segments have the same measurement: $AB = CD$, $AD = BC$, $AC = BD$, $AE = EC$, $BE = ED$, $AE = EB$, $AE = DE$, and $DE = EC$.

b. In Isosceles Trapezoid *RSTU*, these pairs of line segments have the same measurement: $RU = ST$, $RT = SU$, $RV = SV$, and $UV = VT$.

16. a. In Square *WXYZ*, $\angle ZWX = 90°$, $\angle WXY = 90°$, $\angle XYZ = 90°$, $\angle YZW = 90°$, $\angle WAX = 90°$, $\angle XAY = 90°$, $\angle YAZ = 90°$, and $\angle ZAW = 90°$.

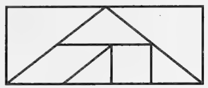
b. In Kite *LMNO*, $\angle OPL = 90°$, $\angle LPM = 90°$, $\angle MPN = 90°$, and $\angle NPO = 90°$.

17.

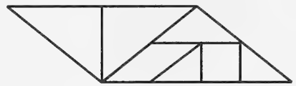
Name of Quadrilateral	Number of Diagonal Lines of Symmetry	Number of Non-Diagonal Lines of Symmetry	Turn Order
Parallelogram	N/A	N/A	2
Rhombus	2	0	2
Rectangle	0	2	2
Square	2	2	4
Trapezoid	N/A	N/A	N/A
Isosceles Trapezoid	0	1	N/A
Trapezium	N/A	N/A	N/A
Kite	1	0	N/A
Dart	1	0	N/A

Now Try This

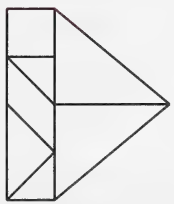
18. You can use a guess, check, and revise strategy to solve this problem. You may also have to change your point of view. **Note:** The parallelogram must be flipped to form each shape.

a. 

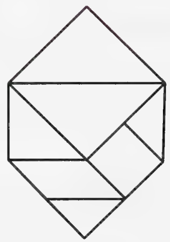
rectangle

b. 

parallelogram

c. 

pentagon

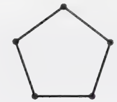
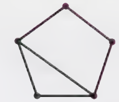
d. 

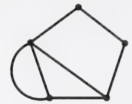
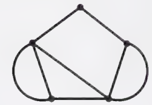
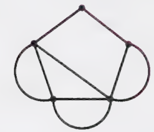
hexagon

Section 1: Activity 4

1. **a.** This network has 4 vertices and 4 edges.
   b. This network has 5 vertices and 8 edges.
   c. This network has 2 vertices and 2 edges.
   d. This network has 4 vertices and 4 edges.
   e. This network has 3 vertices and 6 edges.
   f. This network has 4 vertices and 8 edges.
   g. This network has 10 vertices and 20 edges.

2. Answers will vary. Here are possible answers.

   a.  **b.**  **c.** 

   d.  **e.**  **f.** 

3. **a.** The degree of vertex *B* is 2; vertex *B* is even.
   b. The degree of vertex *B* is 3; vertex *B* is odd.
   c. The degree of vertex *B* is 2; vertex *B* is even.
   d. The degree of vertex *B* is 3; vertex *B* is odd.
   e. The degree of vertex *B* is 4; vertex *B* is even.
   f. The degree of vertex *B* is 4; vertex *B* is even.
   g. The degree of vertex *B* is 4; vertex *B* is even.

4. **a.** Yes, one path is *A*, *B*.
   b. Yes, one path is *A*, *C*, *B*.
   c. Yes, one path is *A*, *B*, *D*, *C*.
   d. Yes, one path is *C*, *D*, *F*, *E*, *A*, *B*, *H*, *G*.

5. **a.** Yes, one path is *A*, *A*.
   b. Yes, one path is *A*, *B*, *C*, *D*, *A*.

6. **a.** yes **b.** yes

7. **Network 1:** no

Network 2: Yes, one path is *A*, *B*, *C*, *D*, *B*.

Network 3: no

Network 4: Yes, one path is *A*, *B*, *C*, *D*, *G*, *C*, *E*, *F*, *G*, *H*, *I*, *D*, *A*.

Network 5: no

Network 6: no

Network 7: Yes, one path is *G*, *H*, *I*, *G*, *D*, *H*, *F*, *I*, *E*, *A*, *D*, *C*, *F*, *B*, *E*, *G*.

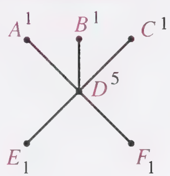
Network 8: no

Network 9: no

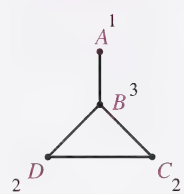
Network 10: Yes, one path is *G*, *A*, *F*, *E*, *A*, *B*, *C*, *D*, *B*, *H*, *G*, *E*, *D*, *H*.

8. a.

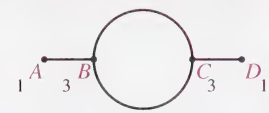
Network 1



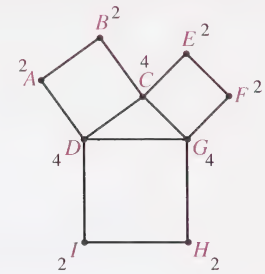
Network 2



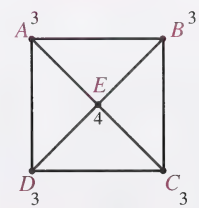
Network 3



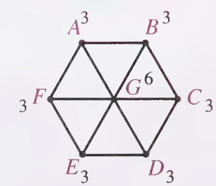
Network 4



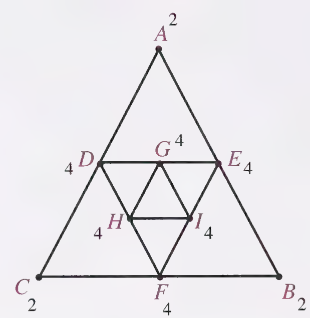
Network 5



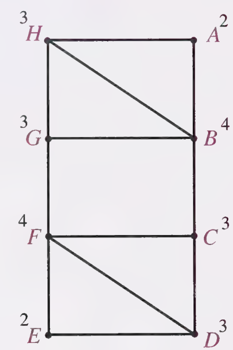
Network 6



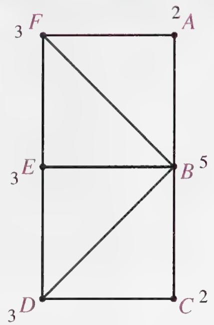
Network 7



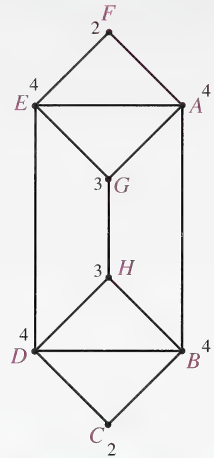
Network 8



Network 9



Network 10



b.

Network	Number of Edges	Sum of the Degrees of the Vertices
1	5	10
2	4	8
3	4	8
4	12	24
5	8	16
6	12	24
7	15	30
8	12	24
9	9	18
10	13	26

c. The sum of the degrees of the vertices in a network is twice the number of edges.

9. a.

Network	Number of Even Vertices	Number of Odd Vertices	Can the Network be Traversed?
1	0	6	no
2	2	2	yes
3	0	4	no
4	9	0	yes
5	1	4	no
6	1	6	no
7	9	0	yes
8	4	4	no
9	2	4	no
10	6	2	yes

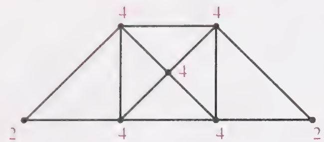
b. Yes, networks 4 and 7 are examples.

c. Yes, networks 2 and 10 are examples.

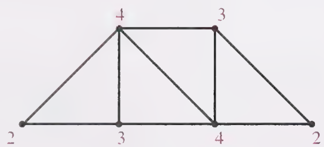
d. No, networks 1, 3, 5, 6, 8, and 9 are examples.

e. When a network has 2 odd vertices, it can be traversed by beginning with one of the odd vertices and ending with the other odd vertex.

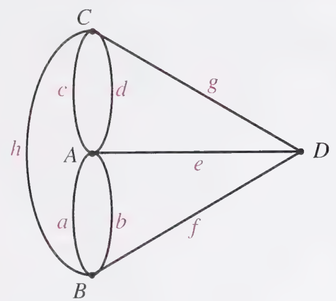
10. **a.** The network has 0 odd vertices. Therefore, the network can be traversed beginning at any vertex.



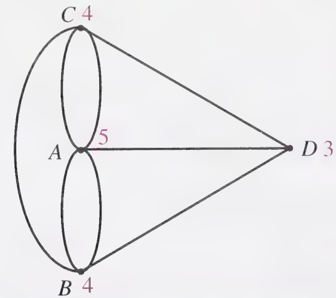
b. The network has 2 odd vertices; therefore, it can be traversed by beginning at one odd vertex and ending at the other odd vertex.



11. **a.**



b. There are 2 odd vertices; therefore, it was now possible to make a round-trip walk crossing all eight bridges only once.



12. **a.**

To / From	A	B	C	D
A	2	2	0	0
B	2	0	1	1
C	0	1	0	3
D	0	1	3	4

b. The sum is 4. This represents the number of trails from A.

c. The sum is 8. This represents the number of trails to D.

d. There are 10 trails on the map. The sum of all the numbers in the table is 20. This number is twice the number of trails because each trail has been counted twice—once entering and once leaving a rest stop.

13. a.

From \ To	A	B	C	D
A	1	1	0	0
B	1	0	1	0
C	0	1	0	2
D	0	1	1	2

b. The sum of the first row is 2. This represents the number of trails from A.

c. The sum of the fourth column is 4. This represents the number of trails to D.

d. The sum of all the numbers in the table is 11. The sum is one more than the number of trails because the trail joining B and C is a two-way trail and it has been counted twice.

14. a. Table 1: No Stopovers

From \ To	A	B	C	D
A	0	1	1	0
B	1	0	0	1
C	0	1	0	0
D	0	1	0	0

b. Table 2: One Stopover

From \ To	A	B	C	D
A	1	1	0	1
B	0	2	1	0
C	1	0	0	1
D	1	0	0	1

c. Table 3: No Stopover or One Stopover

From \ To	A	B	C	D
A	1	2	1	1
B	1	2	1	1
C	1	1	0	1
D	1	1	0	1

d. Each cell in Table 3 is the sum of the corresponding cells in Table 1 and Table 2.

e. No, you cannot travel from one city to every other city with, at most, one stopover. If you travel from City D to City C, two stopovers are needed.

Now Try This

15. You can use a chart to make an organized list and then find the total for each server by adding the numbers in each column.

From \ To	Alice	Barb	Cathy	Dawn	Edwina
Alice	2			1	
Barb	1		3		
Cathy	1	2	1		2
Dawn			2		
Edwina	2			1	
	6	2	6	2	2

Alice gets \$6, Barb gets \$2, Cathy gets \$6, Dawn gets \$2, and Edwina gets \$2.

Section 1: Follow-up Activities

Extra Help

1. a. $s = (n-2)180°$
$= (4-2)180°$
$= (2)180°$
$= 360°$

The sum is 360°.

b. $s = (n-2)180°$
$= (8-2)180°$
$= (6)180°$
$= 1080°$

The sum is 1080°.

c. $s = (n-2)180°$
$= (12-2)180°$
$= (10)180°$
$= 1800°$

The sum is 1800°.

2. a. $360° \div 4 = 90°$

Each angle is 90°.

b. $1080° \div 8 = 135°$

Each angle is 135°.

c. $1800 \div 12 = 150°$

Each angle is 150°.

3. a.

Statement	Reason
$b = 70°$	Opposite angles of a parallelogram are equal.
$a + 70° = 180°$ $\therefore\ a = 110°$	Adjacent angles of a parallelogram are supplementary.
$c = 110°$	Opposite angles of a parallelogram are equal.

b.

Statement	Reason
$x = 140°$	Opposite angles of a parallelogram are equal.
$w + 140° = 180°$ $\therefore\ w = 40°$	Adjacent angles of a parallelogram are supplementary.
$y = 40°$	Opposite angles of a parallelogram are equal.

4. a.

Statement	Reason
$a = 45°$	Inscribed angles subtended by the same arc are equal.

b.

Statement	Reason
$b = 60°$	Inscribed angles subtended by the same arc are equal.

5. a.

Statement	Reason
$a = 2 \times 50°$ $\therefore\ a = 100°$	A central angle is twice the measure of an inscribed angle subtended by the same arc.

b.

Statement	Reason
$\angle ADC = 180°$	It is a straight angle.
$b = \frac{1}{2} \times 180°$ $\therefore\ b = 90°$	An inscribed angle is half the measure of a central angle subtended by the same arc.

Enrichment

1. a.

```
FORWARD 40   or   FD 40   or   REPEAT 3 [FD 40 RT 120]
RIGHT 120         RT 120
FORWARD 40        FD 40
RIGHT 120         RT 120
FORWARD 40        FD 40
RIGHT 120         RT 120
```

$3 \times 120° = 360$

b.

```
FORWARD 40   or   FD 40   or   REPEAT 5 [FD 40 RT 72]
RIGHT 72          RT 72
FORWARD 40        FD 40
RIGHT 72          RT 72
FORWARD 40        FD 40
RIGHT 72          RT 72
FORWARD 40        FD 40
RIGHT 72          RT 72
FORWARD 40        FD 40
RIGHT 72          RT 72
```

$5 \times 72° = 360$

c.

```
FORWARD 30   or   FD 30   or   REPEAT 6 [FD 30 RT 60]
RIGHT 60          RT 60
FORWARD 30        FD 30
RIGHT 60          RT 60
FORWARD 30        FD 30
RIGHT 60          RT 60
FORWARD 30        FD 30
RIGHT 60          RT 60
FORWARD 30        FD 30
RIGHT 60          RT 60
FORWARD 30        FD 30
RIGHT 60          RT 60
```

$6 \times 60° = 360$

d.

FORWARD 30 **or**	FD 30 **or**	REPEAT 8 [FD 30 RT 45]
RIGHT 45	RT 45	
FORWARD 30	FD 30	
RIGHT 45	RT 45	
FORWARD 30	FD 30	
RIGHT 45	RT 45	
FORWARD 30	FD 30	
RIGHT 45	RT 45	$8 \times 45° = 360$
FORWARD 30	FD 30	
RIGHT 45	RT 45	
FORWARD 30	FD 30	
RIGHT 45	RT 45	
FORWARD 30	FD 30	
RIGHT 45	RT 45	
FORWARD 30	FD 30	
RIGHT 45	RT 45	

2. The shape resembles a circle. Increasing the length of a side increases the size of the circle.

Section 2: Activity 1

1. a. $P = 2\ell + 2w$

$= 2(9.2) + 2(5.8)$

$= 18.4 + 11.6$

$= 30.0$

The perimeter is 30 cm.

b. $P = 2\ell + 2w$

$= 2(6.9) + 2(3.2)$

$= 13.8 + 6.4$

$= 20.2$

The perimeter is 20.2 m.

2. a. $P = 2\ell + 2w$

$= 2(4) + 2(3)$

$= 8 + 6$

$= 14$

The perimeter of the room is 14 m.

b. $14 \times 4.79 = 67.06$

The baseboards will cost $67.06.

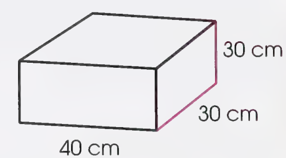
3. **Step 1:** Find the smallest length of tape George could use in one direction. It is equal to the perimeter of the 30 cm × 30 cm rectangular face.

$P = 2\ell + 2w$

$= 2(30) + 2(30)$

$= 60 + 60$

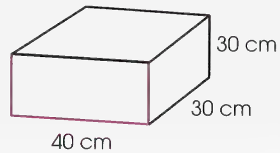
$= 120$



The smallest length in one direction is 120 cm.

Step 2: Find the smallest length of tape George could use in the other direction. It is equal to the perimeter of the $40 \text{ cm} \times 30 \text{ cm}$ rectangular face.

$$\begin{aligned} P &= 2\ell + 2w \\ &= 2(40) + 2(30) \\ &= 80 + 60 \\ &= 140 \end{aligned}$$



The smallest length in the other direction is 140 cm.

Step 3: Find the smallest length in two directions.

$$120 + 140 = 240$$

The smallest length of tape is 260 cm.

4. To find the minimum distance, calculate the outside perimeter of the Pentagon.

$$\begin{aligned} P &= ns \\ &= 5(302) \\ &= 1510 \end{aligned}$$

The person would travel a minimum distance of 1510 m while walking around the outside of the Pentagon.

5. **a.** $$\begin{aligned} P &= ns \\ &= 4(820) \\ &= 3280 \end{aligned}$$

The perimeter of the pasture is 3280 m.

b. $3 \times 3280 = 9840$

The farmer will have to purchase 9840 m of barbwire.

6. $$\begin{aligned} P &= ns \\ &= 36(6.6) \\ &= 237.6 \end{aligned}$$

The perimeter of the first Ferris wheel was 237.6 m.

7. $$\begin{aligned} C &= \pi d \\ &\doteq 3.14(7.1) \\ &\doteq 22.3 \end{aligned}$$

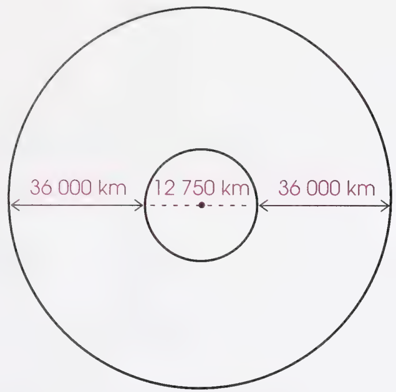
Use a calculator; you may press the π key instead of entering 3.14.

The circumference of the clock face is about 22.3 m.

8. **a.** $$\begin{aligned} C &= \pi d \\ &\doteq 3.14(12\,750) \\ &\doteq 40\,035 \end{aligned}$$

The circumference of Earth is about 40 035 km.

b. Step 1: Make a diagram and calculate the diameter of the orbit.



$$12\,750 + 2(36\,000) = 84\,750$$

The diameter of the orbit is 84 750 km.

Step 2: Calculate the distance travelled in one orbit.

$$\begin{aligned} C &= \pi d \\ &\doteq 3.14(84\,750) \\ &\doteq 266\,115 \end{aligned}$$

The satellite travels about 266 115 km in one orbit.

9. Step 1: Calculate the circumference of the hoop.

$$\begin{aligned} C &= \pi d \\ &\doteq 3.14(45) \\ &\doteq 141.3 \end{aligned}$$

The circumference of the hoop is about 141.3 cm.

Step 2: Calculate the circumference of the basketball.

$$\begin{aligned} C &= \pi d \\ &\doteq 3.14(24.5) \\ &\doteq 76.93 \end{aligned}$$

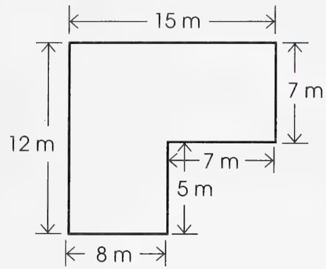
The circumference of the basketball is about 76.93 cm.

Step 3: Calculate the difference and round.

$$\begin{aligned} 141.3 - 76.93 &= 64.37 \\ &\doteq 64 \end{aligned}$$

The difference is about 64 cm.

10. a. Step 1: Make a diagram and determine what segments make up the perimeter.



The perimeter is made up of two segments of 7 m, and one segment each of 15 m, 5 m, 8 m, and 12 m.

Step 2: Calculate the perimeter.

$$\begin{aligned} P &= 2(7)+15+5+8+12 \\ &= 14+15+5+8+12 \\ &= 54 \end{aligned}$$

The perimeter is 54 m.

So, 54 m of fencing is required.

b. Step 1: Calculate the length of the fence that would be put between the swimming pool and the wading pool.

The distance is 8 m.

So, 8 m of fencing is required between the pools.

Step 2: Find the total amount required.

$$54+8=62$$

A total of 62 m of fencing is required altogether.

11. a. Step 1: Determine which segments and curves make up the path Joe travels.

The path is made up of two segments of 175 m, and two semicircles, each with a radius of 20 m.

Step 2: Calculate the circumference of the two semicircles.
Note: Two semicircles make one circle.

$$\begin{aligned} C &= \pi d \\ &\doteq 3.14(40) \\ &\doteq 125.6 \end{aligned}$$

$r = 20$ m
$d = 40$ m

The circumference of the two semicircles is 125.6 m.

Step 3: Calculate the total distance and round.

$$\begin{aligned} P &= 2(175)+125.6 \\ &= 350+125.6 \\ &= 475.6 \\ &\doteq 476 \end{aligned}$$

Joe travels about 476 m in one lap of the inside lane.

b. **Step 1:** Determine which segments and curves make up the perimeter.

The perimeter is made up of two segments of 175 m and two semicircles, each with a radius of 22 m.

Step 2: Calculate the circumference of the two semicircles.
Note: Two semicircles make one circle.

$$C = \pi d$$
$$\doteq 3.14(44)$$
$$\doteq 138.16$$

$r = 22$ m
$d = 44$ m

The circumference of the two semicircles is 138.16 m.

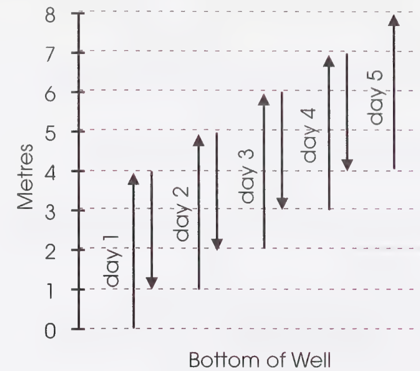
Step 3: Calculate the total distance and round.

$$P = 2(175) + 138.16$$
$$= 350 + 138.16$$
$$= 488.16$$
$$\doteq 488$$

Joe travels about 488 m in one lap of the outside lane.

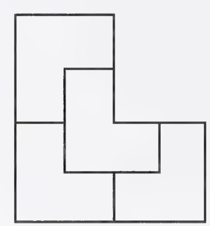
Now Try This

12. You can make a diagram to help you solve the problem.



It will take the spider 5 days to reach the top of the well.

13. You must change your point of view in order to solve the problem. The lots will not be the "usual" rectangular shape. This is one solution.



14. Use logical reasoning to solve the problem.

Step 1: Calculate the number of dimes in the pile.

$$327.06 \div 2.07 = 158$$

There are 158 dimes in the pile.

Step 2: Calculate the value of the pile of coins.

$$158 \times 0.1 = 15.80$$

The pile of coins has a value of $15.80.

Did You Know?

15. **a.** The *Bluenose* was designed by William J. Roué. Construction on the *Bluenose* began after the 1920 International Fishermen's Race; the schooner was launched on March 26, 1921.

b. The *Bluenose* was the fastest fishing schooner ever built. It won five consecutive International Fishermen's Race trophies. These races were the last ever held.

16. **a.** The foresail (4) resembles a trapezoid.
b. The jib topsail (1) resembles a triangle.

17.
$$\begin{aligned} C &= \pi d \\ &\doteq 3.14(1.8) \\ &\doteq 5.7 \end{aligned}$$

The circumference of a dime is about 5.7 cm.

Section 2: Activity 2

1. The rectangle has 12 rows of 16 square units. So, the area is 192 square units.

You know that each unit is 1 cm^2. So, the area is 192 cm^2.

2. **a.**
$$\begin{aligned} A &= \ell w \\ &= 6(4) \\ &= 24 \end{aligned}$$

The area is 24 cm^2.

b.
$$\begin{aligned} A &= \ell w \\ &= 7.5(2.8) \\ &= 21 \end{aligned}$$

The area is 21 m^2.

3.
$$\begin{aligned} A &= \ell w \\ &= 15(12) \\ &= 180 \end{aligned}$$

The area that Margaret rototills is 180 m^2.

4. **Step 1:** Find the area of an average hockey rink.

$$\begin{aligned} A &= \ell w \\ &\doteq 60.6(26) \\ &\doteq 1575.6 \end{aligned}$$

The area of an average hockey rink is about 1575.6 m^2.

Step 2: Find the area of an Olympic hockey rink.

$$\begin{aligned} A &= \ell w \\ &\doteq 60.6(30.3) \\ &\doteq 1836.2 \end{aligned}$$

The area of an Olympic hockey rink is about 1836.2 m^2.

Step 3: Find the difference.

$$1836.2 - 1575.6 \doteq 260.6$$

The Olympic hockey rink is about 260.6 m^2 greater in area.

5. **Step 1:** Find the area of the backyard.

$$\begin{aligned} A &= \ell w \\ &= 30(25) \\ &= 750 \end{aligned}$$

The area of the backyard is 750 m^2.

Step 2: Find the amount of seed needed.

$$750 \div 200 = 3.75$$

The amount of seed needed is 3.75 kg.

6. **Step 1:** Find the area of the opening of the outdoor-soccer goal.

$$\begin{aligned} A &= \ell w \\ &= 7.3(2.4) \\ &= 17.52 \end{aligned}$$

The area of the opening of the outdoor-soccer goal is 17.52 m^2.

Step 2: Find the area of the opening of the indoor-soccer goal.

$$\begin{aligned} A &= \ell w \\ &= 3.8(2) \\ &= 7.6 \end{aligned}$$

The area of the opening of the indoor-soccer goal is 7.6 m^2.

Step 3: Find the area of the opening of the ice-hockey goal.

$$\begin{aligned} A &= \ell w \\ &= 1.8(1.2) \\ &= 2.16 \end{aligned}$$

The area of the opening of the ice-hockey goal is 2.16 m^2.

Step 4: Find the area of the opening of the field-hockey goal.

$$\begin{aligned} A &= \ell w \\ &= 3.7(2.1) \\ &= 7.77 \end{aligned}$$

The area of the opening of the field-hockey goal is 7.77 m^2.

a. $17.52 - 7.6 = 9.92$

The area of the opening of the outdoor-soccer goal is 9.92 m^2 more than the area of the opening of the indoor-soccer goal.

b. $7.77 - 2.16 = 5.61$

The area of the opening of the field-hockey goal is 5.61 m^2 more than the area of the opening of the ice-hockey goal.

7. a. The parallelogram has a base of 14 cm and a height of 12 cm.

b. The new rectangle has a length of 14 cm and a width of 12 cm. These measurements are the same as the base and height of the parallelogram.

c.

$$\begin{aligned} A &= \ell w \\ &= 14(12) \\ &= 168 \end{aligned}$$

1 square unit = 1 cm^2

The area is 168 cm^2.

d. Because the same pieces were used, the area of the parallelogram is equal to the area of the rectangle; therefore, the area of the parallelogram is 168 cm^2.

8. a.

$$\begin{aligned} A &= bh \\ &= 6(8) \\ &= 48 \end{aligned}$$

The area of the parallelogram is 48 cm^2.

b.

$$\begin{aligned} A &= bh \\ &= 7(5) \\ &= 35 \end{aligned}$$

The area of the parallelogram is 35 cm^2.

9.

$$\begin{aligned} A &= bh \\ &= 3.2(1.7) \\ &= 5.44 \\ &\doteq 5.4 \end{aligned}$$

The area of the flight on the dart is about 5.4 cm^2.

10.

$$\begin{aligned} A &= bh \\ &= 14.0(12.3) \\ &= 172.2 \end{aligned}$$

The area of the pane is 172.2 cm^2.

11. **a.** In each triangle, the base is 8 cm and the height is 9 cm.

b. The base is 8 cm and the height is 9 cm.

c.
$$\begin{aligned} A &= bh \\ &= 8(9) \\ &= 72 \end{aligned}$$

1 square unit = 1 cm^2

The area of the parallelogram is 72 cm^2.

d. The area of each triangle is half the area of the parallelogram because two congruent triangles were used.

$$\frac{1}{2} \times 72 = 36$$

The area of each triangle is 36 cm^2.

c.
$$\begin{aligned} A &= \frac{bh}{2} \\ &= \frac{15(6)}{2} \\ &= \frac{90}{2} \\ &= 45 \end{aligned}$$

The area is 45 cm^2.

d.
$$\begin{aligned} A &= \frac{bh}{2} \\ &= \frac{8(6)}{2} \\ &= \frac{48}{2} \\ &= 24 \end{aligned}$$

The area is 24 mm^2.

12. **a.**
$$\begin{aligned} A &= \frac{bh}{2} \\ &= \frac{9(12)}{2} \\ &= \frac{108}{2} \\ &= 54 \end{aligned}$$

The area is 54 cm^2.

b.
$$\begin{aligned} A &= \frac{bh}{2} \\ &= \frac{5(9)}{2} \\ &= \frac{45}{2} \\ &= 22.5 \end{aligned}$$

The area is 22.5 m^2.

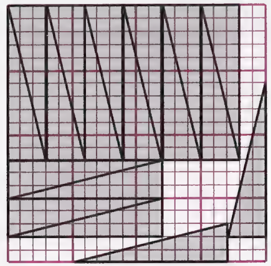
13.
$$\begin{aligned} A &= \frac{bh}{2} \\ &= \frac{60(50)}{2} \\ &= \frac{3000}{2} \\ &= 1500 \end{aligned}$$

The area of this older-style yield sign is 1500 cm^2.

14.
$$\begin{aligned} A &= \frac{bh}{2} \\ &= \frac{7(3.5)}{2} \\ &= \frac{24.5}{2} \\ &= 12.25 \end{aligned}$$

The area of this part of the roof is 12.25 m^2.

15. a. Yes, 18 pennants will fit on the material.



Sketches may vary. Here is one example. **Note:** Each square on the graph paper represents 5 cm × 5 cm of material.

b. Step 1: Find the area of 1 pennant.

$$\begin{aligned} A &= \frac{bh}{2} \\ &= \frac{60(15)}{2} \\ &= \frac{900}{2} \\ &= 450 \end{aligned}$$

The area of 1 pennant is 450 cm^2.

Step 2: Find the area of 18 pennants.

$$450 \times 18 = 8100$$

The area of 18 pennants is 8100 cm^2.

To make 18 pennants, 8100 cm^2 of felt is used.

16. a. In each figure one base is 4 cm long and one base is 8 cm. The height of each figure is 9 cm.

b. The base is 12 cm long and the height is 9 cm. The length of each base of the parallelogram is equal to the sum of the lengths of base_1 and base_2 of the trapezoids.

c.

$$\begin{aligned} A &= bh \\ &= 12(9) \\ &= 108 \end{aligned}$$

1 square unit = 1 cm^2

The area is 108 cm^2.

d. The area of each trapezoid is half the area of the parallelogram because two congruent trapezoids were used.

$$\frac{1}{2} \times 108 = 54$$

The area of each trapezoid is 54 cm^2.

17. a.

$$\begin{aligned} A &= \frac{(b_1 + b_2)h}{2} \\ &= \frac{(12+34)11}{2} \\ &= \frac{(46)11}{2} \\ &= \frac{506}{2} \\ &= 253 \end{aligned}$$

The area is 253 cm^2.

b.

$$\begin{aligned} A &= \frac{(b_1 + b_2)h}{2} \\ &= \frac{(5.2+6.0)4.8}{2} \\ &= \frac{(11.2)4.8}{2} \\ &= \frac{53.76}{2} \\ &= 26.88 \end{aligned}$$

The area is 26.88 cm^2.

c. $$\begin{aligned} A &= \frac{(b_1 + b_2)h}{2} \\ &= \frac{(12+15)9}{2} \\ &= \frac{(27)9}{2} \\ &= \frac{243}{2} \\ &= 121.5 \end{aligned}$$

The area is 121.5 m^2.

d. $$\begin{aligned} A &= \frac{(b_1 + b_2)h}{2} \\ &= \frac{(9.5+23.8)15}{2} \\ &= \frac{(33.3)15}{2} \\ &= \frac{499.5}{2} \\ &= 249.75 \end{aligned}$$

The area is 249.75 m^2.

18. $$\begin{aligned} A &= \frac{(b_1 + b_2)h}{2} \\ &= \frac{(40+30)20}{2} \\ &= \frac{(70)20}{2} \\ &= \frac{1400}{2} \\ &= 700 \end{aligned}$$

The area is 700 cm^2.

19. $$\begin{aligned} A &= \frac{(b_1 + b_2)h}{2} \\ &= \frac{(2.5+3.5)2}{2} \\ &= \frac{(6)2}{2} \\ &= \frac{12}{2} \\ &= 6 \end{aligned}$$

The area is 6 m^2.

20. **a.** **Step 1:** Measure the diameter.

The diameter is 14 cm.

Step 2: Calculate the circumference.

$$\begin{aligned} C &= \pi d \\ &\doteq 3.14(14) \\ &\doteq 43.96 \end{aligned}$$

The circumference is about 43.96 cm.

b. The base of the new "parallelogram" is about 22.5 cm; it is about half of the circumference. The height of the new "parallelogram" is about 7 cm; it is equal to the length of the radius.

c. $A = bh$

$= 22.5(7)$

$= 157.5$

The area of the "parallelogram" is about 157.5 cm^2.

d. The area of the circle is equal to the area of the "parallelogram" because all the pieces were used; therefore, the area is about 157.5 cm^2.

21. **a.** $A = \pi r^2$

$\doteq 3.14(2^2)$

$\doteq 3.14(4)$

$\doteq 12.56$

$\doteq 12.6$

$\doteq 13$

The area is about 13 m^2.

b. $r = 14$ mm

$A = \pi r^2$

$\doteq 3.14(14^2)$

$\doteq 3.14(196)$

$\doteq 615.44$

$\doteq 615.4$

$\doteq 615$

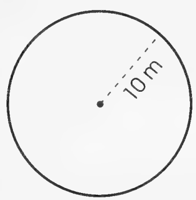
The area is about 615 mm^2.

22. $A = \pi r^2$

$\doteq 3.14(10^2)$

$\doteq 3.14(100)$

$\doteq 314$



The total area that the horse can eat is about 314 m^2.

23. $A = \pi r^2$

$\doteq 3.14(25^2)$

$\doteq 3.14(625)$

$\doteq 1962.5$

$\doteq 1963$

The total area that can be watered is about 1963 m^2.

24. **Step 1:** Find the area of one side of a dime.

$A = \pi r^2$

$\doteq 3.14(0.9^2)$

$\doteq 3.14(0.81)$

$\doteq 2.5434$

$\doteq 2.5$

$d = 1.8$ cm
$r = 0.9$ cm

The area of one side of a dime is about 2.5 cm^2.

Step 2: Find the area of one side of a quarter.

$A = \pi r^2$

$\doteq 3.14(1.2^2)$

$\doteq 3.14(1.44)$

$\doteq 4.5216$

$\doteq 4.5$

$d = 2.4$
$r = 1.2$

The area of one side of a quarter is about 4.5 cm^2.

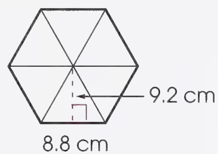
Step 3: Find the difference.

$$4.5 - 2.5 = 2.0$$

The area of one side of a quarter is about $2.0\ \text{cm}^2$ greater than the area of one side of a dime.

25. **a.** **Step 1:** Divide the hexagon into 6 congruent triangles and find the area of one triangle.

$$\begin{aligned} A &= \frac{bh}{2} \\ &= \frac{8.8 \times 9.2}{2} \\ &= \frac{80.96}{2} \\ &= 40.48 \end{aligned}$$



The area of one triangle is $40.48\ \text{cm}^2$.

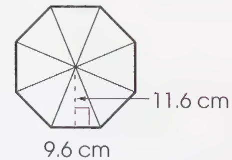
Step 2: Find the area of the entire regular hexagon.

$$6 \times 40.48 = 242.88$$

The area of the regular hexagon is $242.88\ \text{cm}^2$.

b. **Step 1:** Divide the octagon into 8 congruent triangles and find the area of one triangle.

$$\begin{aligned} A &= \frac{bh}{2} \\ &= \frac{9.6 \times 11.6}{2} \\ &= \frac{111.36}{2} \\ &= 55.68 \end{aligned}$$



The area of one triangle is $55.68\ \text{cm}^2$.

Step 2: Find the area of the entire regular octagon.

$$8 \times 55.68 = 445.44$$

The area of the regular octagon is $445.44\ \text{cm}^2$.

26. **a.**

$$\begin{aligned} A &= \frac{nsa}{2} \\ &= \frac{5(1.6)(1.1)}{2} \\ &= \frac{8.8}{2} \\ &= 4.4 \end{aligned}$$

The area is $4.4\ \text{m}^2$.

b.

$$\begin{aligned} A &= \frac{nsa}{2} \\ &= \frac{6(5.6)(4.8)}{2} \\ &= \frac{161.28}{2} \\ &= 80.64 \end{aligned}$$

The area is $80.64\ \text{cm}^2$.

27. $A = \dfrac{nsa}{2}$

$$= \frac{8(16.5)(20)}{2}$$

$$= \frac{2640}{2}$$

$$= 1320$$

The area is $1320\ \text{cm}^2$.

28. $A = \dfrac{nsa}{2}$

$$\doteq \frac{11(7)(13)}{2}$$

$$\doteq \frac{1001}{2}$$

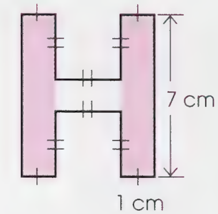
$$\doteq 500.5$$

The area is about $500.5\ \text{mm}^2$.

29. **a.** **Step 1:** Determine what familiar shapes make up the composite figure.

The composite figure is made up of two congruent rectangles and another rectangle.

Step 2: Calculate the area of the two congruent rectangles.



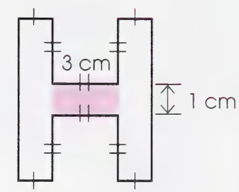
$$A = \ell w$$
$$= 7(1)$$
$$= 7$$

Each rectangle has an area of $7\ \text{cm}^2$.

$$2 \times 7 = 14$$

Two rectangles have an area of $14\ \text{cm}^2$.

Step 3: Calculate the area of the other rectangle.



$$A = \ell w$$
$$= 3(1)$$
$$= 3$$

The area of the other rectangle is $3\ \text{cm}^2$.

Step 4: Calculate the total area.

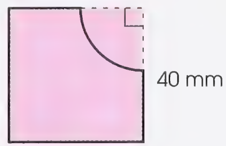
$$14 + 3 = 17$$

The total area of the figure is $17\ \text{cm}^2$.

b. Step 1: Determine what familiar shapes make up the composite figure.

The composite figure is a square with part of a circle removed.

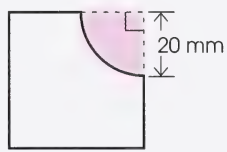
Step 2: Calculate the area of the square.



$$A = s^2$$
$$= 40^2$$
$$= 1600$$

The area of the square is 1600 mm^2.

Step 3: Calculate the area of the circular part that has been removed. **Note:** The circular part is one-fourth of a circle with a radius of 20 mm.



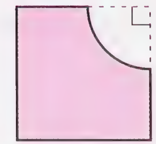
$$A = \pi r^2$$
$$\doteq (3.14)(20^2)$$
$$\doteq (3.14)(400)$$
$$\doteq 1256$$

The area of a circle with a radius of 20 mm is 1256 mm^2.

$$\frac{1}{4}(1256) = 314$$

The area of the circular part is about 314 mm^2.

Step 4: Find the difference in the areas.



$$1600 - 314 \doteq 1286$$

The area of the figure is about 1286 mm^2.

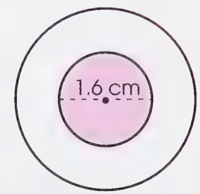
30. Step 1: Determine what familiar shapes make up the composite figure.

The composite figure is a circle with a smaller circle removed.

Step 2: Find the area of the inside circle.

$$A = \pi r^2$$
$$\doteq 3.14(0.8^2)$$
$$\doteq 3.14(0.64)$$
$$\doteq 2.0$$

$d = 1.6$ cm
$r = 0.8$ cm

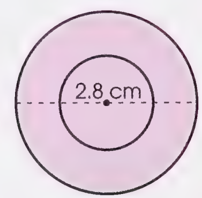


The area of the inside circle is about 2.0 cm^2.

Step 3: Find the area of the entire front of the coin.

$$A = \pi r^2$$
$$\doteq 3.14(1.4^2)$$
$$\doteq 3.14(1.96)$$
$$\doteq 6.2$$

$d = 2.8$ cm
$r = 1.4$ cm



The area of the front of the coin is about 6.2 cm^2.

Step 4: Find the area of the outer ring.

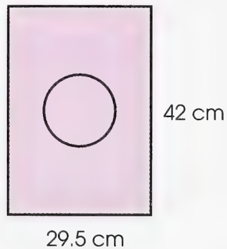
$$6.2 - 2.0 \doteq 4.2$$

The area of the outer ring is about 4.2 cm^2.

31. a. Step 1: Determine what familiar shapes make up the sail of the kite.

The sail of the kite is a rectangle with a circular region removed.

Step 2: Find the area of the rectangle.



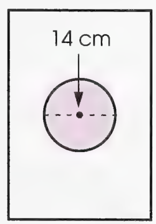
$$\begin{aligned} A &= \ell w \\ &= 42(29.5) \\ &= 1239 \end{aligned}$$

The area of the rectangle is 1239 cm^2.

Step 3: Find the area of the circular hole.

$$\begin{aligned} A &= \pi r^2 \\ &\doteq 3.14(7^2) \\ &\doteq 153.86 \end{aligned}$$

$d = 14$ cm
$r = 7$ cm

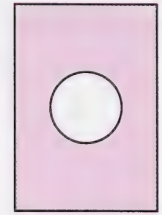


The area of the circular hole is about 153.86 cm^2.

Step 4: Find the difference in areas.

$$1239 - 153.86 \doteq 1085.14$$

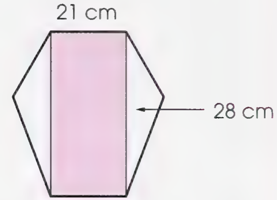
The area of the sail of the Korean Fighter kite is about 1085.14 cm^2.



b. Step 1: Determine what familiar shapes make up the sail of the Sled kite.

The sail of the kite is a rectangle and two congruent triangles.

Step 2: Find the area of the rectangle.

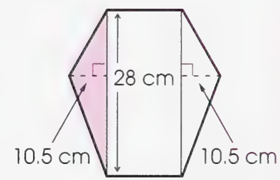


$$\begin{aligned} A &= \ell w \\ &= 28(21) \\ &= 588 \end{aligned}$$

The area of the rectangle is 588 cm^2.

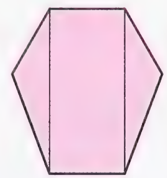
Step 3: Find the area of one triangle.

$$\begin{aligned} A &= \frac{bh}{2} \\ &= \frac{28(10.5)}{2} \\ &= \frac{294}{2} \\ &= 147 \end{aligned}$$



The area of one triangle is 147 cm^2.

Step 4: Find the total area.

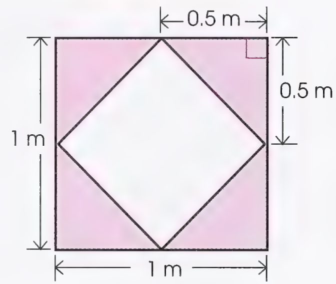


$$588+2(147)=588+294$$
$$=882$$

The area of the sail of the Sled kite is $882\ \text{cm}^2$.

Now Try This

32. To solve this problem, you must change your point of view. You can cover the corners of the window with four congruent triangles.



$$A=\frac{bh}{2}$$
$$=\frac{0.5(0.5)}{2}$$
$$=0.125$$

The area of each triangle is $0.125\ \text{m}^2$.

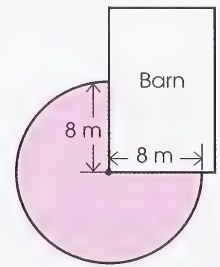
$$4\times 0.125=0.5$$

The area of the four triangles is $0.5\ \text{m}^2$, which is half the window area.

This leaves a square opening that is 1 m across and 1 m from top to bottom. **Note:** The measurements are of the diagonals of the square opening.

33. **a.** **Step 1:** Make a diagram to help you understand the problem.

The goat can graze on a region that is three-fourths of a circle with a radius of 8 m.



Step 2: Find the area of the region that is three-fourths of a circle with a radius of 8 m.

$$A=\pi r^2$$
$$\doteq 3.14\left(8^2\right)$$
$$\doteq 3.14(64)$$
$$\doteq 200.96$$

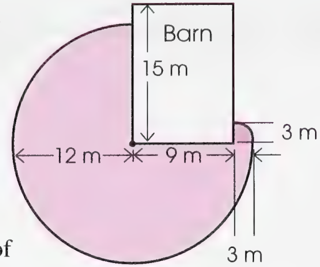
$$\frac{3}{4}(200.96)\doteq 150.72$$

$\frac{3}{4}=0.75$

The area is about $150.72\ \text{m}^2$.

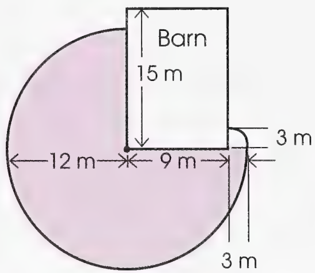
The goat can graze on about $151\ \text{m}^2$.

b. Step 1: Make a diagram or a model to help you understand the problem.



The goat can graze on the region that is three-fourths of a circle with a radius of 12 m, plus the region that is one-fourth of a circle with a radius of 3 m.

Step 2: Find the area of the region that is three-fourths of a circle with a radius of 12 m.



$$
\begin{aligned}
A &= \pi r^2 \\
&\doteq 3.14\left(12^2\right) \\
&\doteq 3.14(144) \\
&\doteq 452.16
\end{aligned}
$$

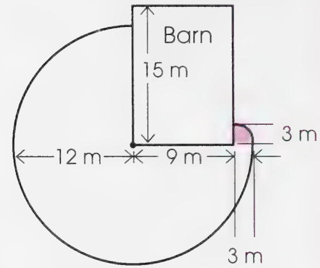
$\frac{3}{4} \times 452.16 \doteq 339.12$

$\frac{3}{4} = 0.75$

The area is about 339.12 m^2.

Step 3: Find the area of the region that is one-fourth of a circle with a radius of 3 m.

$$
\begin{aligned}
A &= \pi r^2 \\
&\doteq 3.14\left(3^2\right) \\
&\doteq 3.14(9) \\
&\doteq 28.26
\end{aligned}
$$



$\frac{1}{4} \times 28.26 \doteq 7.065$

$\frac{1}{4} = 0.25$

The area is about 7.065 m^2.

Step 4: Find the total area on which the goat can graze.

$$
\begin{aligned}
339.12 + 7.065 &\doteq 346.185 \\
&\doteq 346
\end{aligned}
$$

The total area is about 346 m^2.

The goat can graze on about 346 m^2.

Section 2: Follow-up Activities

Extra Help

1. a. $$\begin{aligned} P &= ns \\ &= 4(2) \\ &= 8 \end{aligned}$$

   The perimeter of the square is 8 cm.

   b. $$\begin{aligned} P &= ns \\ &= 6(1) \\ &= 6 \end{aligned}$$

   The perimeter of the hexagon is 6 cm.

   c. The circumference of the circle is between 6 cm and 8 cm.

   d. $$\begin{aligned} C &= \pi d \\ &\doteq 3.14(2) \\ &\doteq 6.28 \end{aligned}$$

   The circumference of the circle is about 6.28 cm.

2. a. $$\begin{aligned} A &= s^2 \\ &= 2^2 \\ &= 4 \end{aligned}$$

   The area of the larger square is 4 cm^2.

   b. $$\begin{aligned} A &= s^2 \\ &= 1.4^2 \\ &= 1.96 \end{aligned}$$

   The area of the smaller square is 1.96 cm^2.

   c. The area of the circle is between 1.96 cm^2 and 4 cm^2.

   d. $$\begin{aligned} A &= \pi r^2 \\ &\doteq 3.14\left(1^2\right) \\ &\doteq 3.14 \end{aligned}$$

   $d = 2 \text{ cm}$
   $r = 1 \text{ cm}$

   The area of the circle is about 3.14 cm^2.

3. a. A parallelogram is formed.

   b. As you pull, the base of the new shape remains the same.

   c. As you pull, the height of the new shape decreases.

   d. As you pull, the area of the new shape decreases. The area decreases as the height decreases.

4. a. **Step 1:** Estimate the area. Use the formula for a triangle.

Rounding

$$A = \frac{bh}{2} \doteq \frac{5 \times 3}{2} \doteq 7.5$$

The area is about $7.5\ \text{cm}^2$.

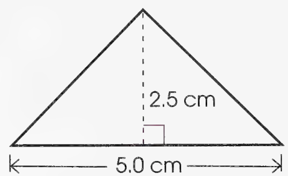
Front-end Digits

$$A = \frac{bh}{2} \doteq \frac{5 \times 2}{2} \doteq 5$$

The area is about $5\ \text{cm}^2$.

Step 2: Calculate the area.

$$A = \frac{bh}{2} = \frac{5.0 \times 2.5}{2} = 6.25$$



The area is $6.25\ \text{cm}^2$.

Step 3: Compare the calculated answer and the estimate to decide if your answer is reasonable.

$6.25 \doteq 7.5$ $\quad$ $6.25 \doteq 5$

Therefore, the answer is reasonable.

The area is $6.25\ \text{cm}^2$.

b. **Step 1:** Estimate the area. Use the formula for a triangle.

Rounding

$$A = \frac{bh}{2} \doteq \frac{3 \times 3}{2} \doteq 4.5$$

The area is about $4.5\ \text{cm}^2$.

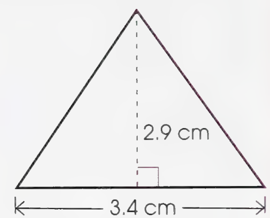
Front-end Digits

$$A = \frac{bh}{2} \doteq \frac{3 \times 2}{2} \doteq 3$$

The area is about $3\ \text{cm}^2$.

Step 2: Calculate the area.

$$A = \frac{bh}{2} = \frac{3.4 \times 2.9}{2} = 4.93$$



The area is $4.93\ \text{cm}^2$.

Step 3: Compare the calculated answer and the estimate to decide if your answer is reasonable.

$4.93 \doteq 4.5$ $\quad$ $4.93 \doteq 3$

Therefore, the answer is reasonable.

The area is $4.93\ \text{cm}^2$.

c. **Step 1:** Estimate the area. Use the formula for a square.

Rounding

$$A = s^2$$
$$\doteq 4^2$$
$$\doteq 16$$

The area is about 16 cm^2.

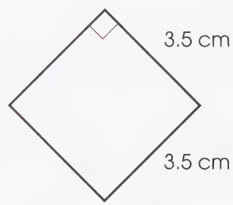
Front-end Digits

$$A = s^2$$
$$\doteq 3^2$$
$$\doteq 9$$

The area is about 9 cm^2.

Step 2: Calculate the area.

$$A = s^2$$
$$= (3.5)^2$$
$$= 12.25$$



The area is 12.25 cm^2.

Step 3: Compare the calculated answer and the estimate to decide if your answer is reasonable.

$12.25 \doteq 16$ $\quad$ $12.25 \doteq 9$

Therefore, the answer is reasonable.

The area is 12.25 cm^2.

d. **Step 1:** Estimate the area. Use the formula for a parallelogram.

Rounding

$$A = bh$$
$$\doteq 3 \times 3$$
$$\doteq 9$$

The area is about 9 cm^2.

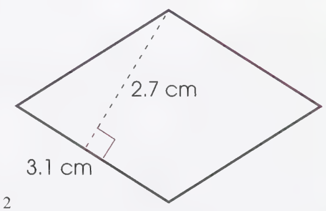
Front-end Digits

$$A = bh$$
$$\doteq 3 \times 2$$
$$\doteq 6$$

The area is about 6 cm^2.

Step 2: Calculate the area.

$$A = bh$$
$$= 3.1 \times 2.7$$
$$= 8.37$$



The area is 8.37 cm^2.

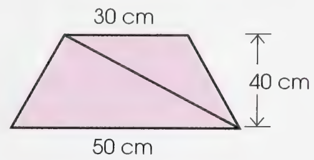
Step 3: Compare the calculated answer and the estimate to decide if your answer is reasonable.

$8.37 \doteq 9$ $\quad$ $8.37 \doteq 6$

Therefore, the answer is reasonable.

The area is 8.37 cm^2.

5. **Step 1:** Find the area of the given side of the top section.



$$A = \frac{bh}{2} = \frac{50(40)}{2} = \frac{2000}{2} = 1000$$

The area of one triangle is 1000 cm^2.

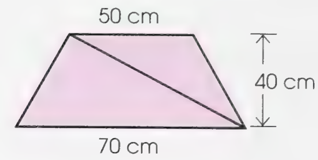
$$A = \frac{bh}{2} = \frac{30(40)}{2} = \frac{1200}{2} = 600$$

The area of the other triangle is 600 cm^2.

$$1000 + 600 = 1600$$

The area of the top section is 1600 cm^2.

Step 2: Find the area of the given side of the middle section.



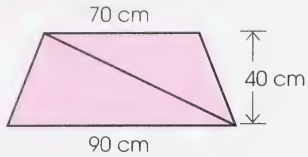
$$A = \frac{bh}{2} = \frac{70(40)}{2} = \frac{2800}{2} = 1400$$

$$A = \frac{bh}{2} = \frac{50(40)}{2} = \frac{2000}{2} = 1000$$

$$1400 + 1000 = 2400$$

The area of the middle section is 2400 cm^2.

Step 3: Find the area of the given side of the bottom section.



$$A = \frac{bh}{2} = \frac{90(40)}{2} = \frac{3600}{2} = 1800$$

$$A = \frac{bh}{2} = \frac{70(40)}{2} = \frac{2800}{2} = 1400$$

$$1800 + 1400 = 3200$$

The area of the bottom section is 3200 cm^2.

Enrichment

1. **a.** You multiply by 1000. **b.** You divide by 1000.

2. **a.** 5 m = 500 cm **b.** 80 m = 8000 cm
   c. 65 mm = 6.5 cm **d.** 0.7 m = 70 cm
   e. 45 mm = 4.5 cm **f.** 173 mm = 17.3 cm

3. **a.** 200 cm = 2 m **b.** 3 km = 3000 m
   c. 8000 mm = 8 m **d.** 53.8 cm = 0.538 m
   e. 5.9 km = 5900 m **f.** 732 mm = 0.732 m

4. **a.** You multiply by 1 000 000. **b.** You divide by 1 000 000.

5. **a.** 250 m^2 = 2 500 000 cm^2 **b.** 0.5 m^2 = 5000 cm^2
   c. 60 mm^2 = 0.6 cm^2 **d.** 850 mm^2 = 8.5 cm^2
   e. 3 m^2 = 30 000 cm^2 **f.** 9000 mm^2 = 90 cm^2

6. **a.** 80 000 cm^2 = 8 m^2 **b.** 6000 cm^2 = 0.6 m^2
   c. 3 km^2 = 3 000 000 m^2 **d.** 8000 mm^2 = 0.008 m^2
   e. 50 km^2 = 50 000 000 m^2 **f.** 550 mm^2 = 0.000 55 m^2

7. **Step 1:** Find the area of the room.

$$A = s^2 = 9^2 = 81$$

The area is 81 m^2.

Step 2: Find the area of one tile in square metres.

$$A = s^2 = 0.3^2 = 0.09$$

$$30 \text{ cm} = 0.3 \text{ m}$$

The area is 0.09 m^2.

Step 3: Find the number of tiles.

$$81 \div 0.09 = 900$$

A total of 900 tiles will be needed.

8. $A = \ell w$

$= 5(0.8)$

$= 4$

$80 \text{ cm} = 0.8 \text{ m}$

The area is 4 m^2.

9. $A = \ell w$

$= 7820(0.0072)$

$= 56.304$

$\doteq 56$

$7.2 \text{ m} = 0.0072 \text{ km}$

The area is about 56 km^2.

10. **a.** Calculate the area of the field.

$A = \ell w$

$= 5(4)$

$= 20$

The area is 20 km^2.

b. **Step 1:** Calculate the area of the field in square metres.

$20 \text{ km}^2 = 20\,000\,000 \text{ m}^2$

The area is $20\,000\,000 \text{ m}^2$.

Step 2: Calculate the number of prairie dogs per square metre.

$$40\,000\,000 \div 20\,000\,000 = 2$$

There are 2 prairie dogs per square metre.

The Game of Trivial Pursuit®

Inventors: Scott Abbott, born in 1949, and Chris Haney, born in 1950, two Montreal journalists

Date: 1979

Significance: Trivial Pursuit became the most commercially successful board game since the invention of Monopoly™ (1935) and SCRABBLE® Brand Crossword Game (1953).

Profile: "Why can't we invent a game as good as this?" Chris Haney, then photo editor of the Montreal *Gazette*, asked his buddy as they settled down at a kitchen table in Montreal on the night of Dec. 15, 1979, for a game of Scrabble.

"What should it be about?" replied Scott Abbott, then a sports reporter for Canadian Press. "How about trivia?" Within an hour, the two players had come up with the basic design of Trivial Pursuit. Less than six months later, Haney quit his job to devote himself to developing the idea. It took a year, much of it spent in Spain, for the co-inventors, now joined by Chris's brother John, to think up the 6 000 questions of which the original game consists. They formed a company, Horn Abbott Limited ("Horn" is Chris Haney's nickname), rented office space in Niagara-on-the-Lake, Ontario, obtained copyrights and patents, went deeply into debt—and had their game ready for market tests in a few Ontario stores by November 1981.

Orders came slowly at first, but during their first year in business the novice entrepreneurs sold 100 000 copies. During their second year they sold 2.4 million copies in Canada and signed an agreement with the manufacturer and distributor of Scrabble for the distribution of Trivial Pursuit in the United States. By 1984 Trivial Pursuit had earned almost a billion dollars in worldwide retail sales. Thirty-four people joined the inventors in raising $75 000 in initial capital. Investment in Trivial Pursuit shares turned out to be more remunerative than investment in any other enterprise during the past ten years.

How is Trivial Pursuit played? In the original version of Trivial Pursuit, known as the Genus edition, players answer questions in one of six categories—Geography, Entertainment, History, Art and Literature, Science and Nature, and Sports and Leisure—determined by the roll of a die. Six questions, and the answers to them, are printed on each of 1000 cards. Correctly answering a question permits a player to move a token with the object of reaching the middle of the board. Once there, the player must correctly answer a final question in a category chosen by his or her opponents.[1]

[1] Sean McCutcheon, "Discoveries and Inventions," *Horizon Canada*, vol. 3, no. 35 (1985): third cover.

The Fishing Schooner *Bluenose*

Designer: William J. Roué, self-taught yacht designer, born in Halifax, N.S., in 1879, and died in 1970

Date: 1920–21

Significance: *Bluenose* was the fastest fishing schooner ever built and became a symbol of national pride in Canada. Its image is found on one side of the 10-cent piece.

Profile: The Grand Banks, off Canada's east coast, was the richest cod-fishing grounds in the world. Here, for three centuries, fishermen in wooden sailing ships hooked cod and preserved them in salt. Saltbankers, as the fishing schooners were known, became obsolete at about the time of World War One; they could not compete economically against steam-powered iron vessels, which were faster, stronger, larger, and safer. *Bluenose*, the fastest saltbanker ever built—one of the fastest ships that ever sailed, in fact—was built when the era of sail was ending, for reasons of pride and for sport rather than for profit.

In 1920 the owner of the *Halifax Herald* newspaper offered the International Fishermen's Trophy to the fastest ship from the deep-sea fishing fleets of Gloucester, Massachusetts, and of Lunenburg, Nova Scotia. In the 1920 race, the U.S. entry beat Canada's. Chagrined, Halifax business interests commissioned William Roué to design a fishing schooner specifically for competition in the races. Although he had no formal training as a marine architect, Roué was a prolific designer of several types of vessels. In this case, the result was *Bluenose*. It cost \$35 000 to build at a time when the average schooner cost \$25 000. It was launched on March 26, 1921, at Lunenburg.

After a first season fishing on the Banks, *Bluenose* sailed to victory in the 1921 schooner race, regaining the trophy, a massive cup, for Canada. The International Fishermen's Race was held four more times. In all these races *Bluenose*, skippered by Angus Walters, represented Canada and never lost.

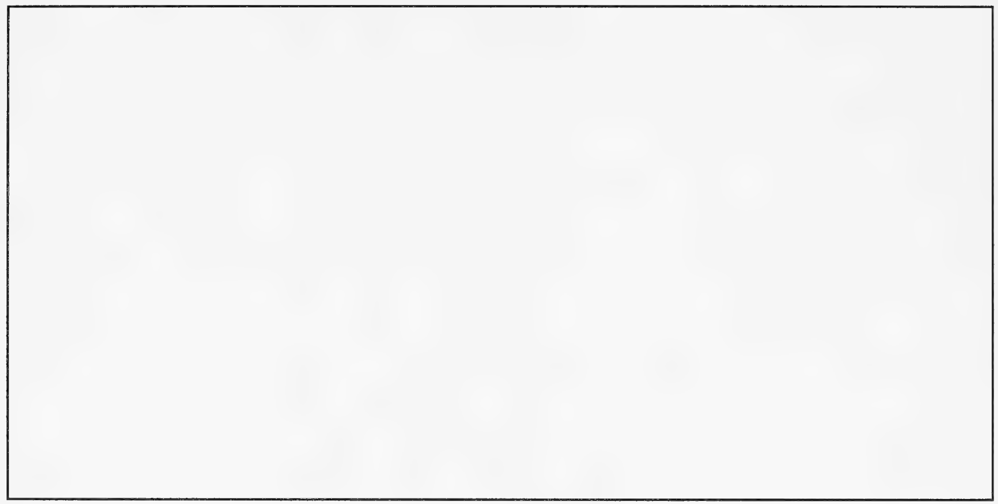
Bluenose became a celebrated ship, but its skipper and crew could not live on glory; they had to make a living by catching cod—a dangerous and unrewarding occupation. In 1926, for instance, *Bluenose* was struck by a "grandfather sea"—the skipper's phrase—and almost wrecked on Sable Island, the Atlantic graveyard. As prices for fish fell in the "hungry thirties," life became increasingly difficult for them. Angus Walters, who had become the sole owner of the aging and unprofitable *Bluenose*, reluctantly sold his ship to the West Indian Trading Company in 1940. On January 28, 1946, *Bluenose* struck a coral reef off Haiti. Her crew escaped but the schooner disappeared.

"Bluenose" is a term for Nova Scotians coined by American loyalists in the 18th century. It refers to the effect of winter cold on human noses.[1]

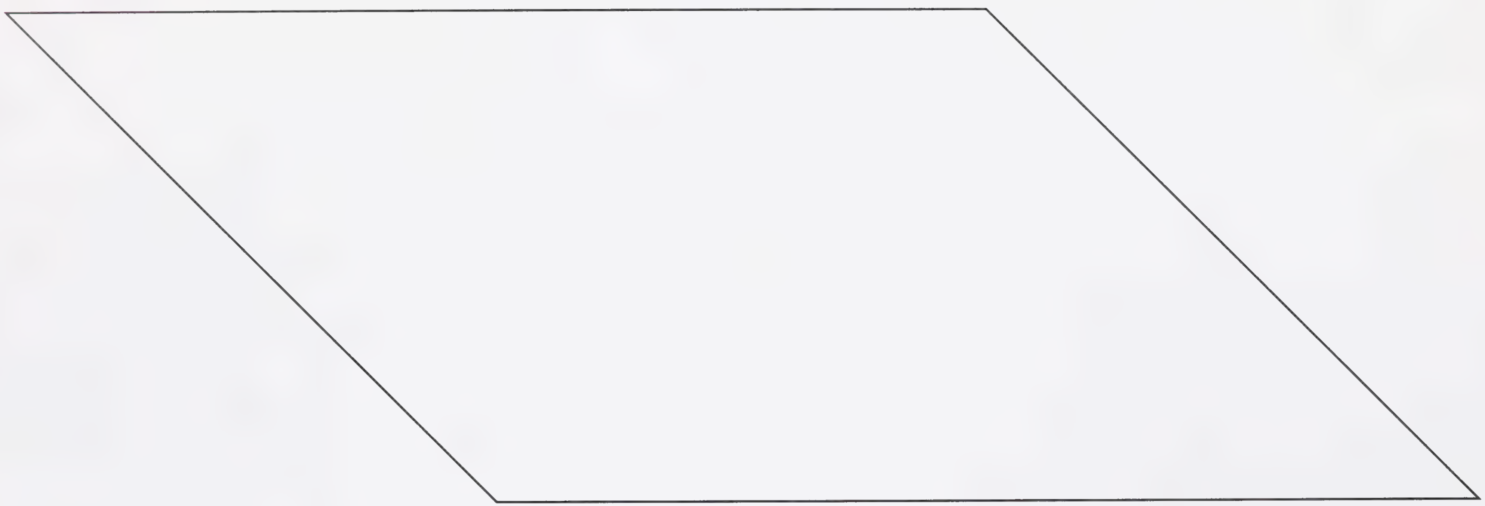
[1] Sean McCutcheon, "Discoveries and Inventions," *Horizon Canada*, vol. 3, no. 36 (1985): third cover.

Dot Paper

Tangram Puzzle 1

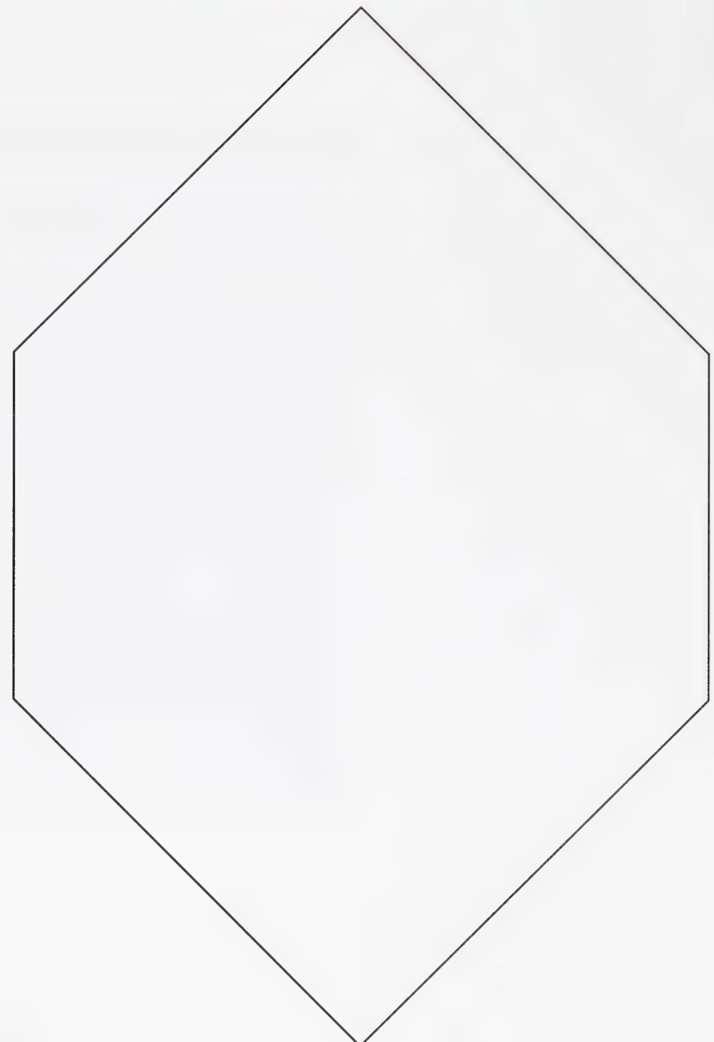


Tangram Puzzle 2

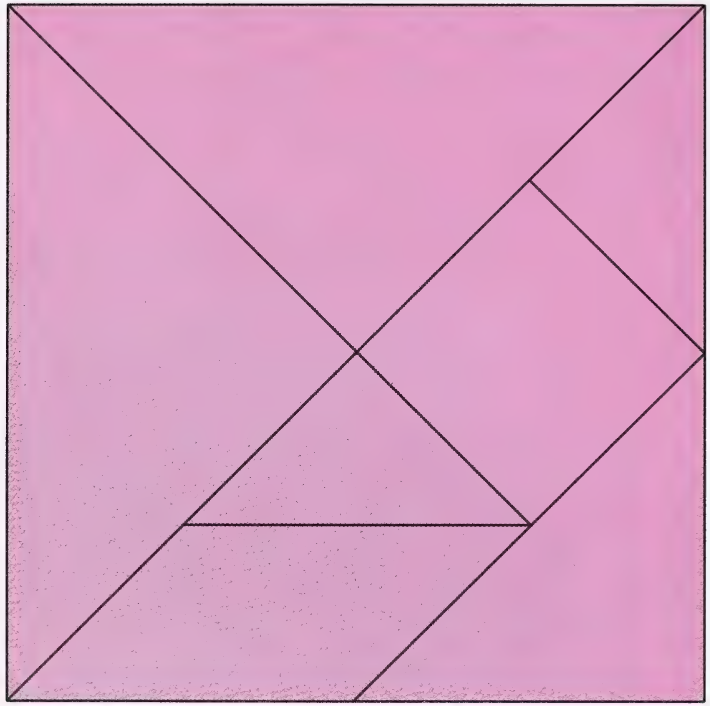


Tangram Puzzle 3

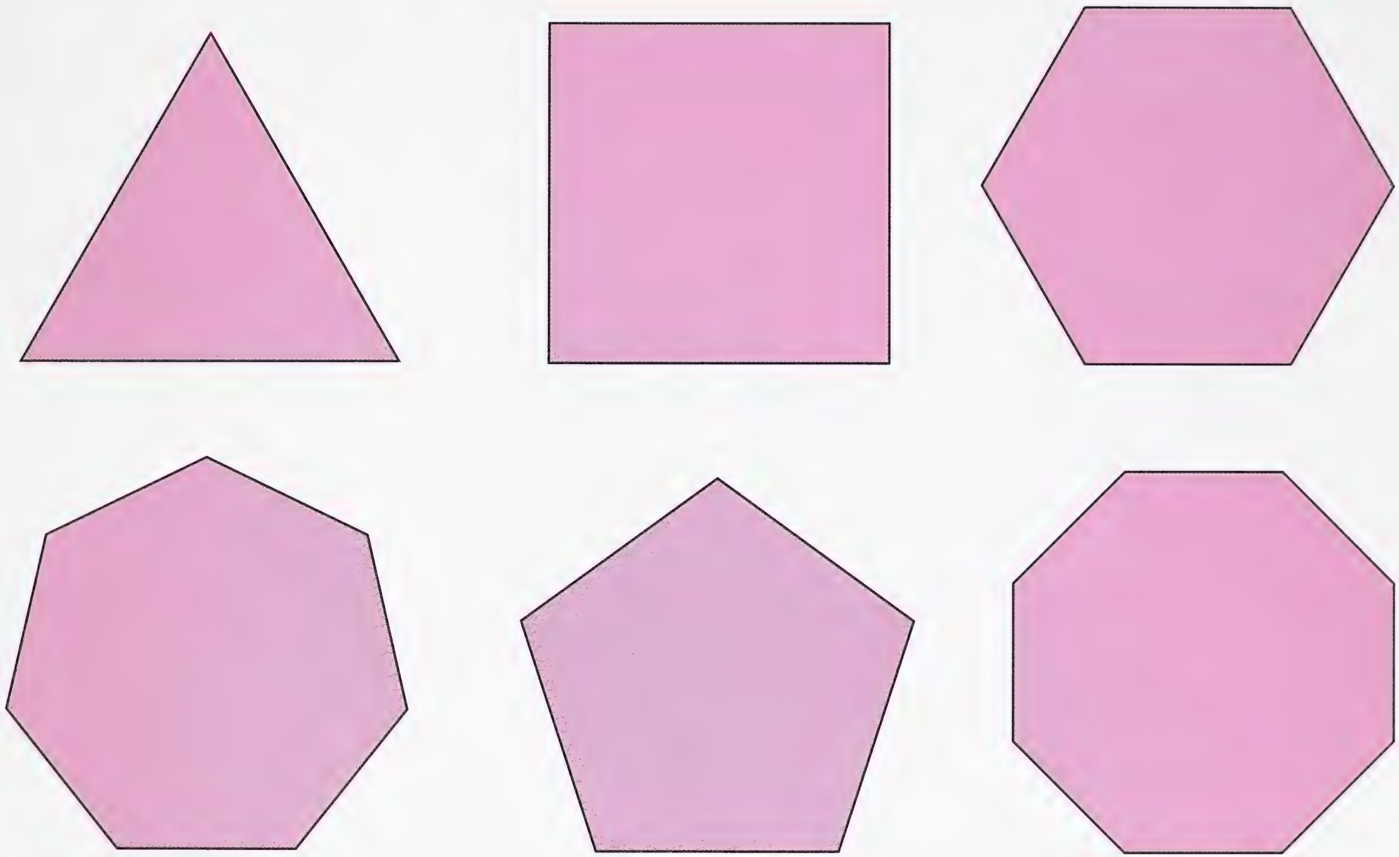
Tangram Puzzle 4



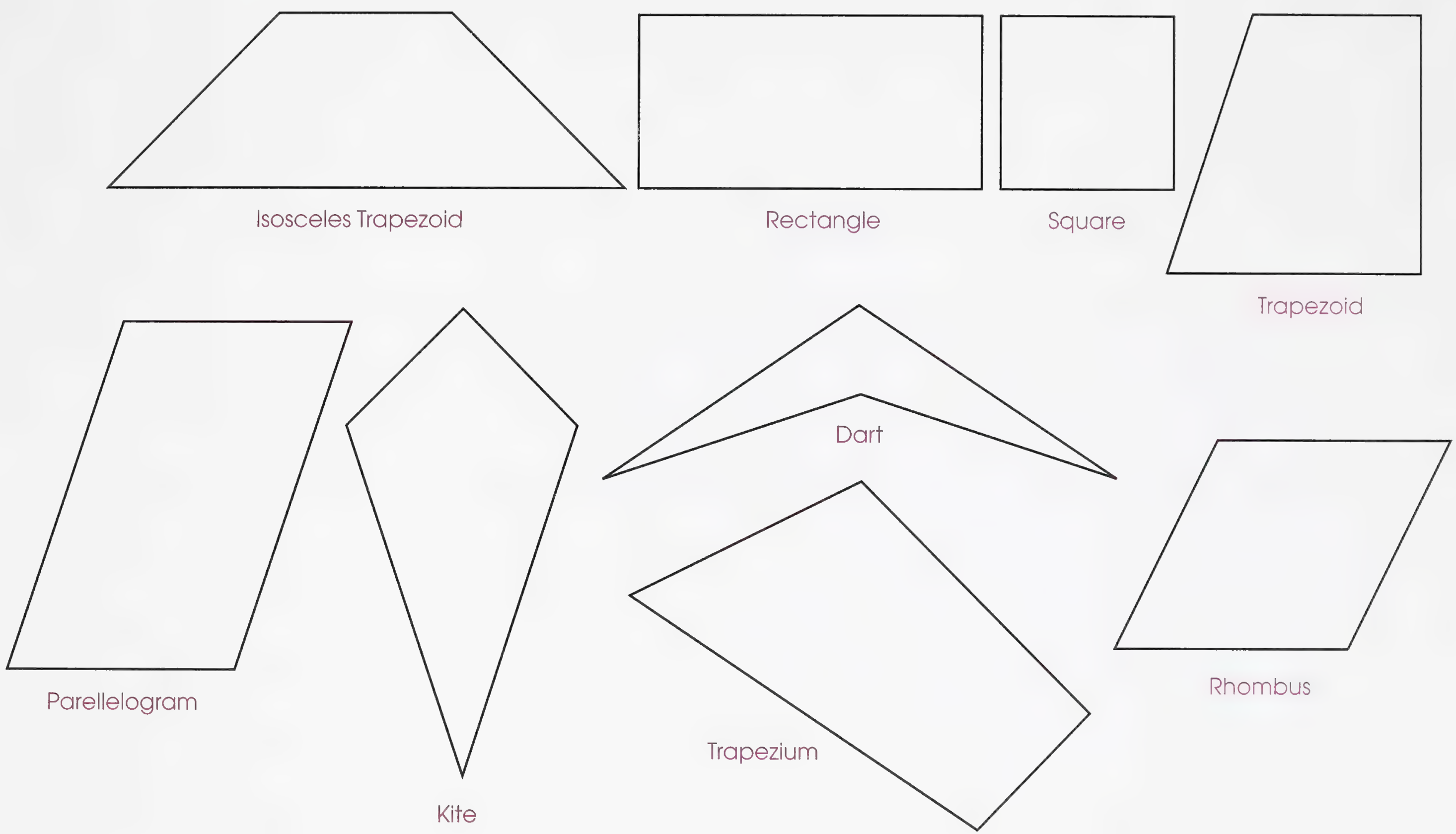
Tangram



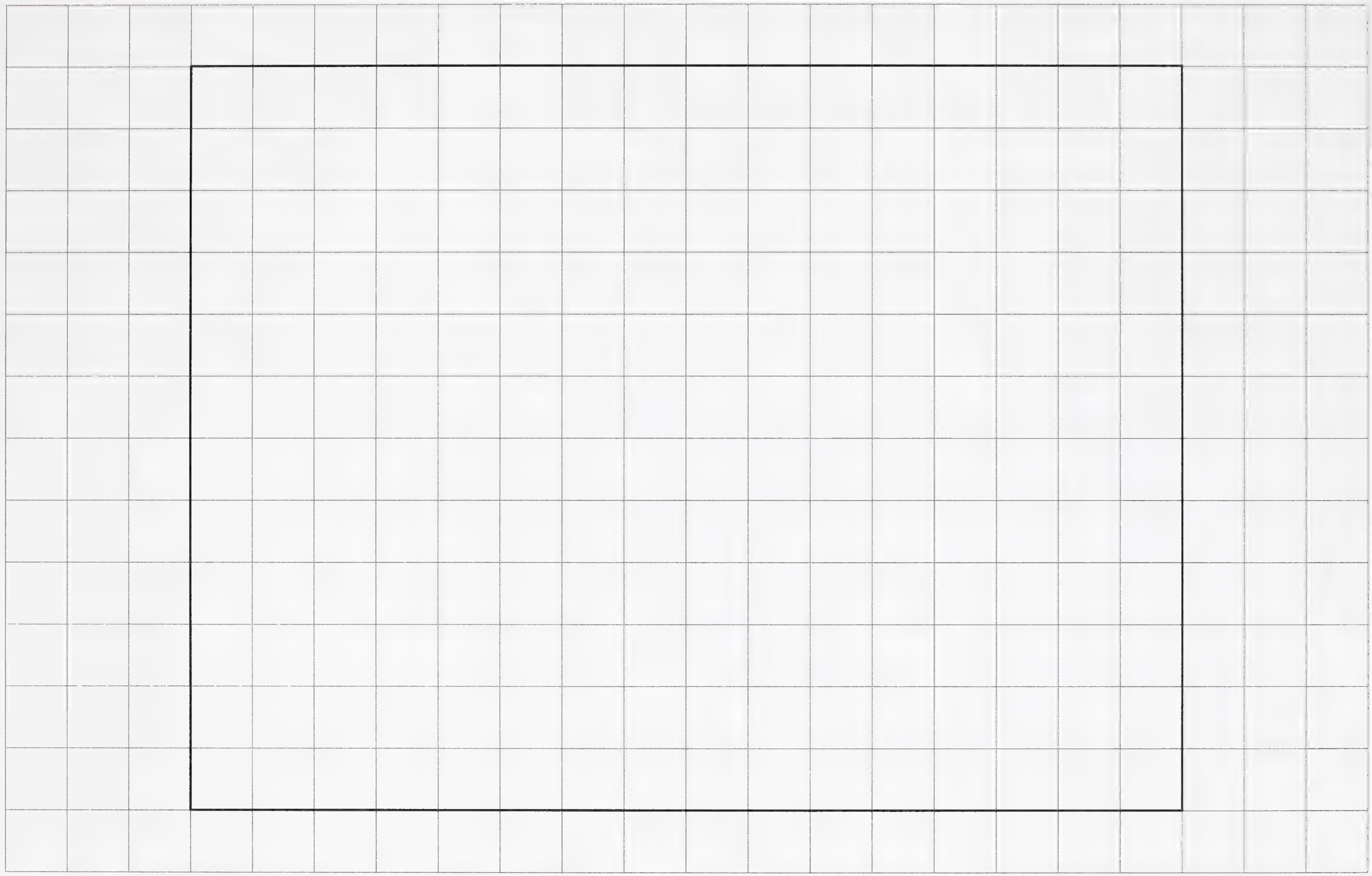
Regular Polygons



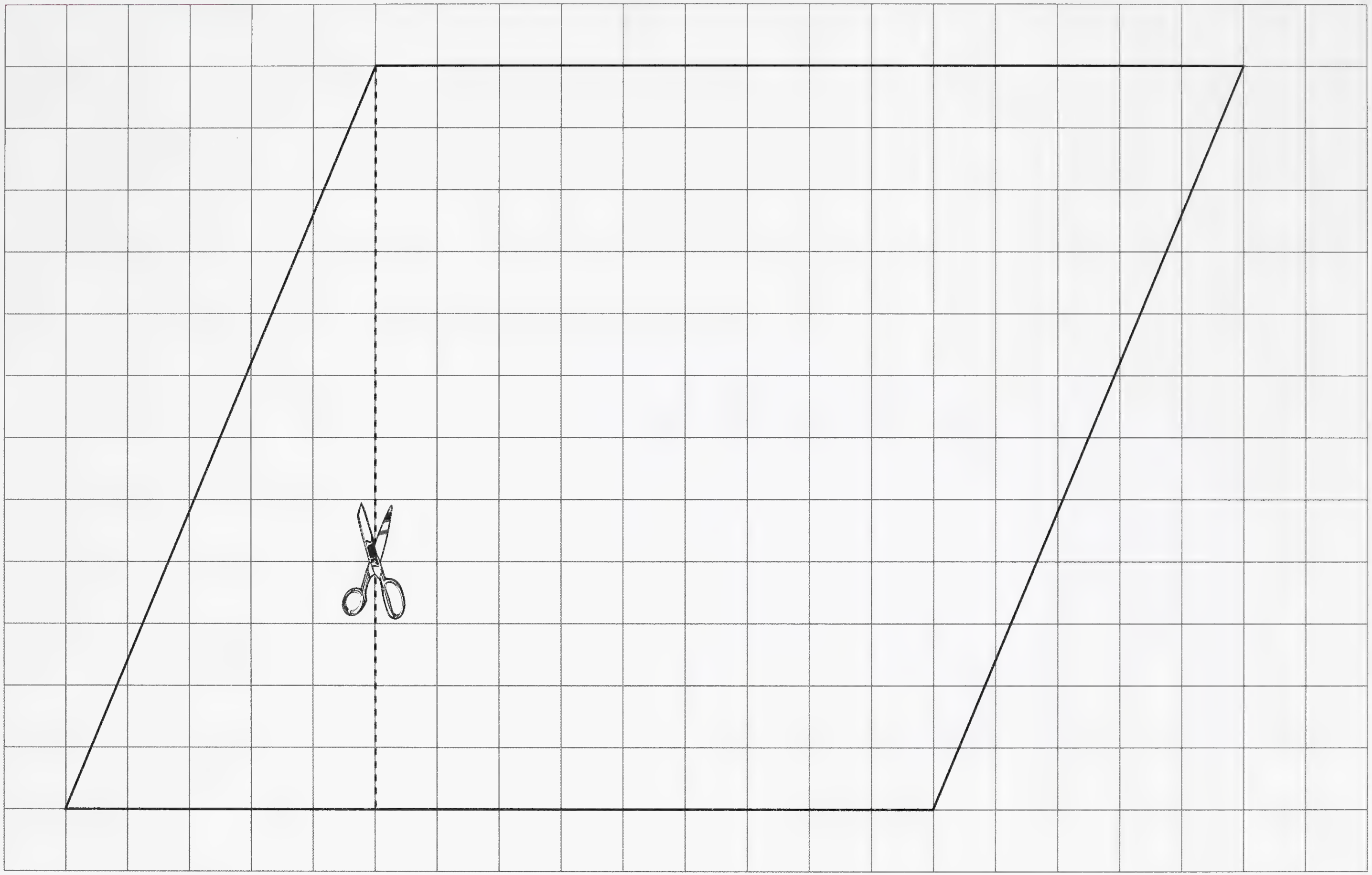
Quadrilaterals



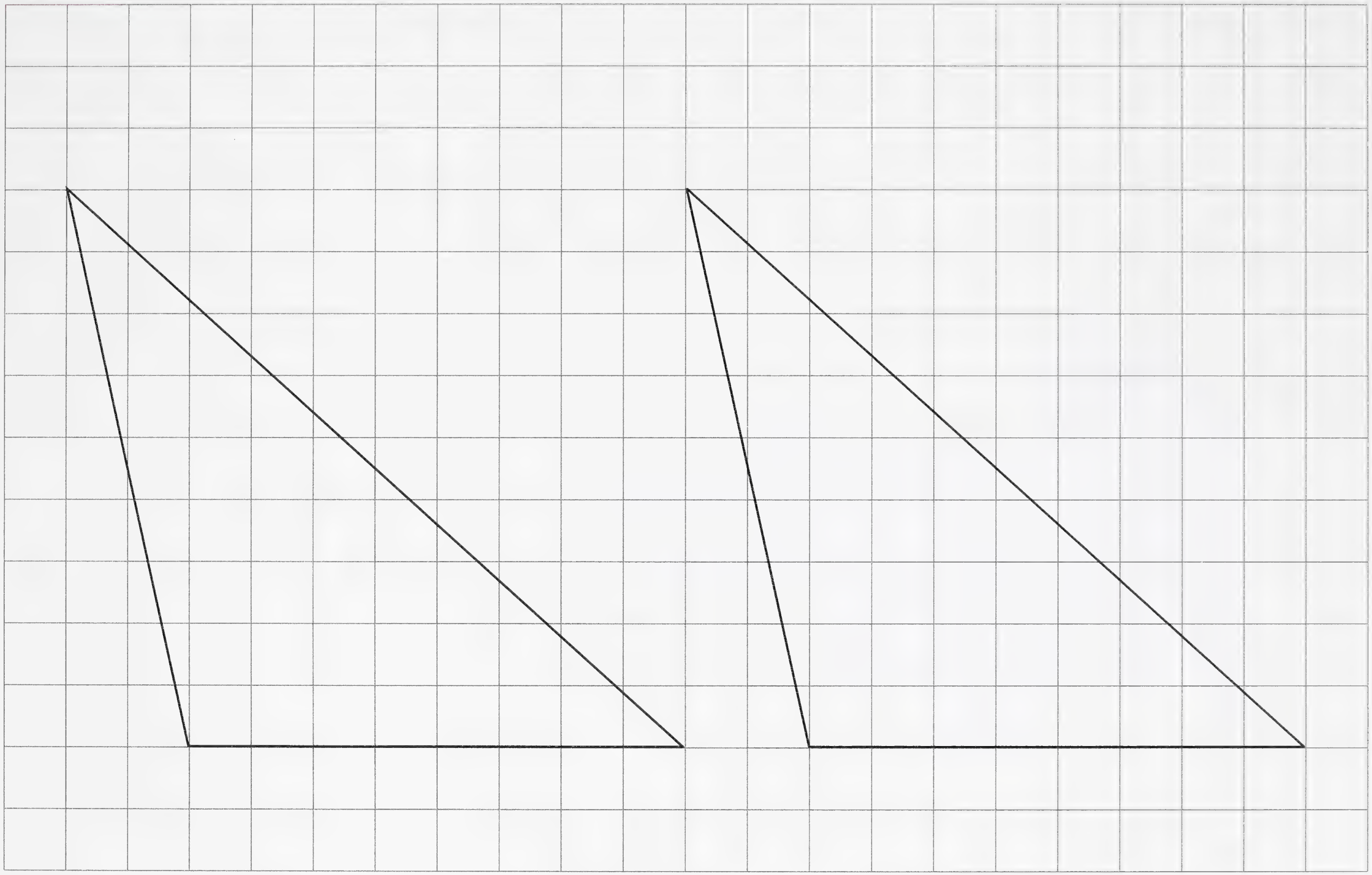
Rectangle



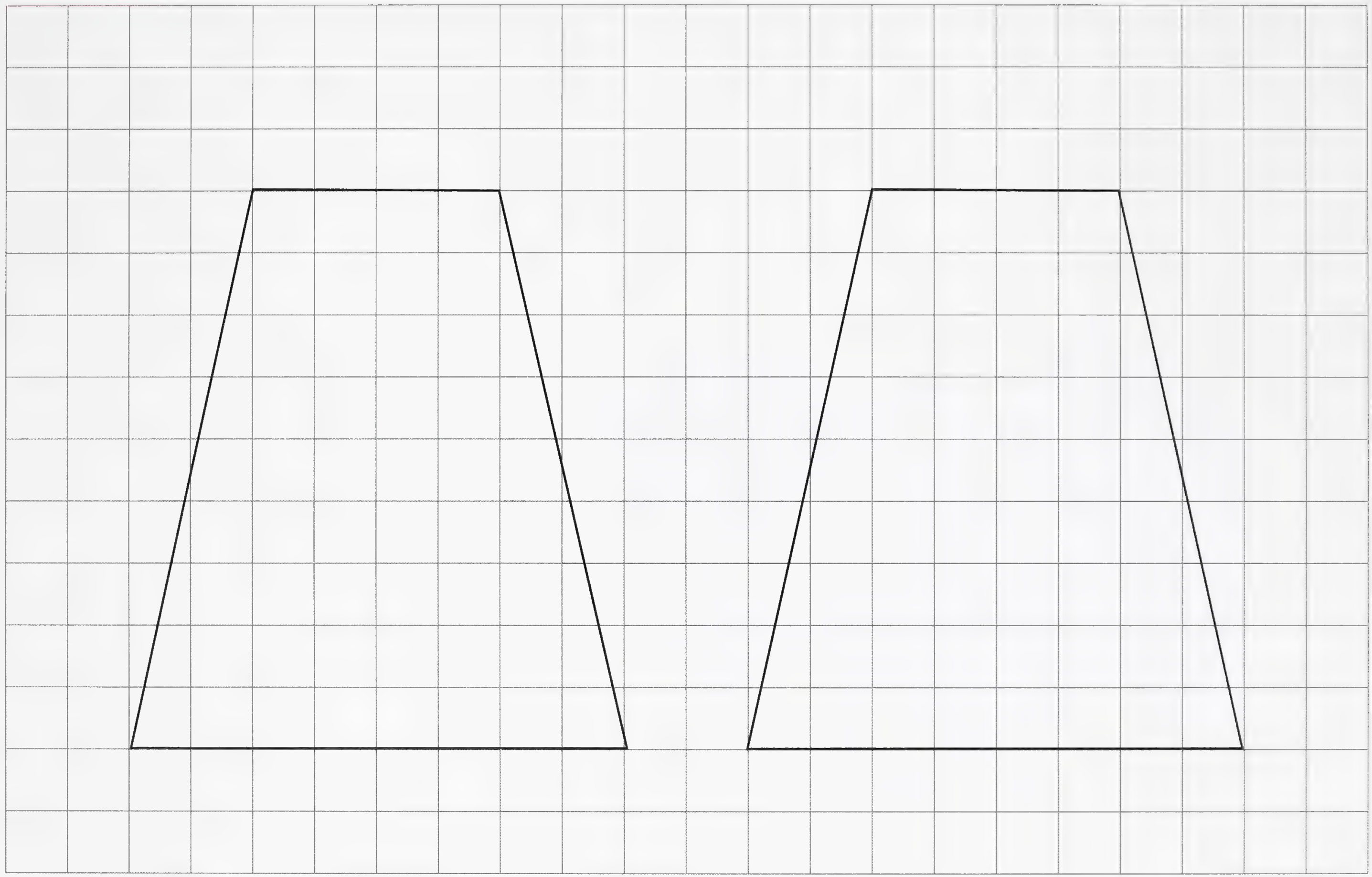
Parallelogram



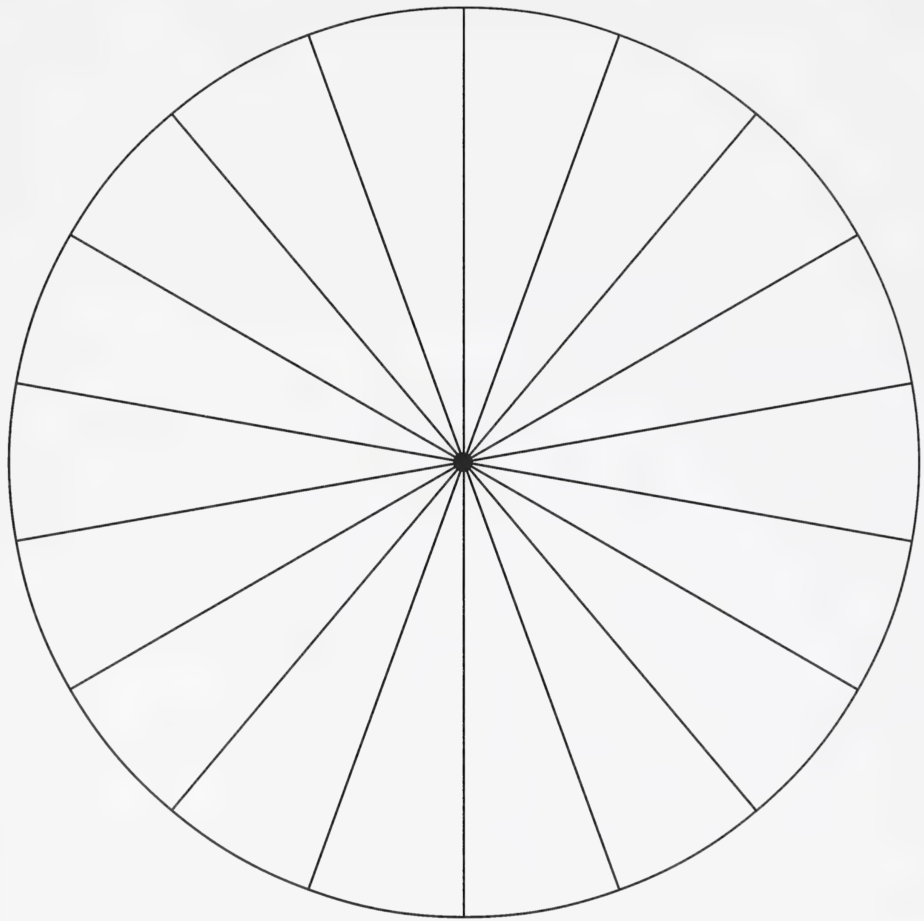
Triangles



Trapezoids



Circle



Student and teacher: Use this cover sheet for mailing or faxing.

ASSIGNMENT BOOKLET

8110 Mathematics 8
Module 6

FOR STUDENT USE ONLY

Date Module Submitted: ____________

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File Number: ____________

Module Number: ____________

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- Has your work been reread to ensure accuracy in spelling and details?
- Is the booklet cover filled out and the correct module label attached?

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Mathematics 8

Module 6

Three-Dimensional Geometry

Assignment Booklet

Distance Learning

Alberta EDUCATION

FOR TEACHER'S USE ONLY

Summary

	Total Possible Marks	Your Mark
Section 1 Assignment	40	
Section 2 Assignment	45	
Final Module Assignment	15	
	100	

Teacher's Comments

This document is intended for	
Students	✓
Teachers	✓
Administrators	
Parents	
General Public	
Other	

Mathematics 8
Assignment Booklet
Module 6
Three-Dimensional Geometry
Learning Technologies Branch
ISBN 0-7741-1405-3

ASSIGNMENT BOOKLET
MATHEMATICS 8 – MODULE 6: THREE-DIMENSIONAL GEOMETRY

Your mark on this module will be determined by how well you do your assignments in this booklet.

Work slowly and carefully. If you are having difficulties, go back and review the appropriate section.

There are two section assignments and one final module assignment in this Assignment Booklet. The total value of these assignments is 100 marks. The value of each assignment is stated in the left margin.

Be sure to proofread each assignment carefully.

40

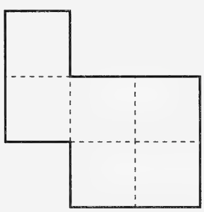
Section 1 Assignment: Properties of Three-Dimensional Shapes

Read all the parts of your assignment carefully and record your answers in the appropriate place.

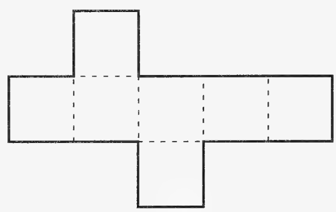
Circle the letter of the best answer for questions 1 to 4.

2 **1.** Which of the following nets can be used to make a cube?

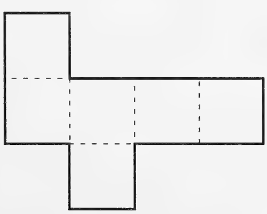
A.



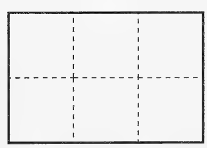
B.



C.



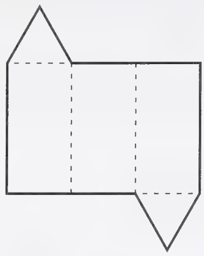
D.



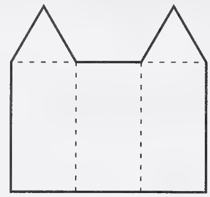
2

2. Which of the following nets can be used to make a triangular prism?

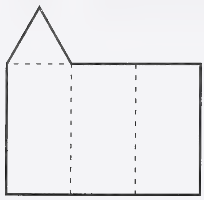
A.



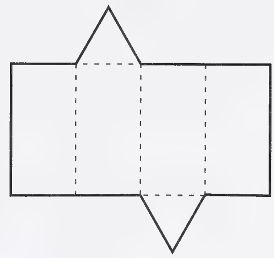
B.



C.



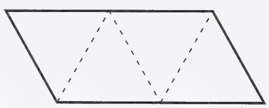
D.



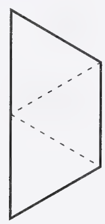
2

3. Which of the following nets can be used to make a triangular pyramid?

A.



B.



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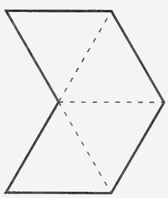
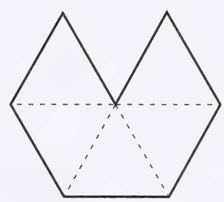


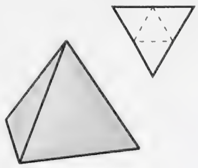
D.



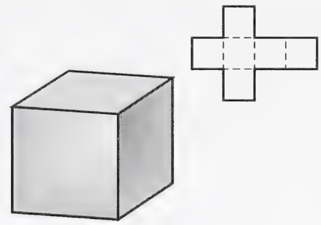
2

4. Following are the sketches of various polyhedra. Beside each is a sketch of the polyhedron's net. The solid of which polyhedron can **not** be used as a die? **Hint:** A die must be completely symmetrical so that each face has an equal opportunity to land face up.

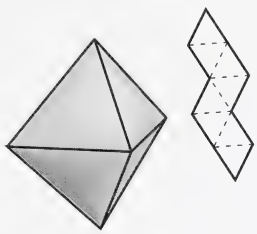
A.



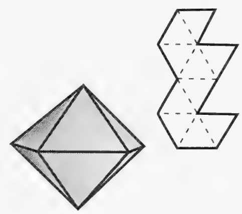
B.



C.

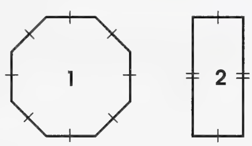


D.



2

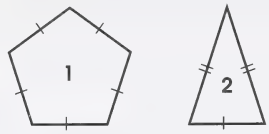
5. a. Name the polyhedron that has polygon 1 as a base and polygon 2 as a "side" face.



2

b. Make a sketch of the net of the polyhedron that you identified in question 5.a.

2 **6.** **a.** Name the polyhedron that has polygon 1 as a base and polygon 2 as a "side" face.



2 **b.** Make a sketch of the net of the polyhedron identified in question 6.a.

Clearly show how you arrived at your answers in questions 7, 8, and 9.

2 **7.** The skeleton of a convex polyhedron is made with toothpicks joined together with mini-marshmallows. If 17 toothpicks and 12 mini-marshmallows are used, how many faces does the polyhedron have?

8. The skeleton of a right prism with a base of 7 sides is made with toothpicks joined together with mini-marshmallows.

2 **a.** How many toothpicks are needed?

2 **b.** How many mini-marshmallows are needed?

9. The skeleton of a pyramid is made with toothpicks and mini-marshmallows. The base of the pyramid has 10 sides.

2 **a.** How many toothpicks are needed?

2 **b.** How many mini-marshmallows are needed?

2 **10. a.** If you make a cross section **perpendicular** to the base of a solid cylinder, what shape is the cross section?

2 **b.** If you make a cross section **parallel** to the base of a solid cylinder, what shape is the cross section?

2 **11. a.** If you make a cross section **perpendicular** to the base of a solid square pyramid, what shape is the cross section?

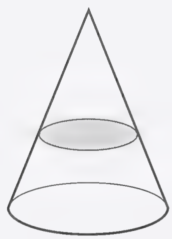
2 **b.** If you make a cross section **parallel** to the base of a square pyramid, what shape is the cross section?

2

12. What do you notice about the cross sections of a solid sphere?

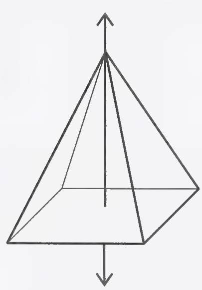
2

13. Is this shaded plane a plane of symmetry for a cone? Explain why or why not.



2

14. Is this line an axis of symmetry for a square pyramid? Explain why or why not.



45

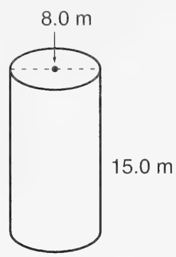
Section 2 Assignment: Measurement of Three-Dimensional Shapes

Read all the parts of your assignment carefully and record your answers in the appropriate place. Clearly show how you arrived at your answers.

1. For each of the following solids calculate the volume and the surface area. **Note:** Round each answer to the nearest tenth.

6

a.



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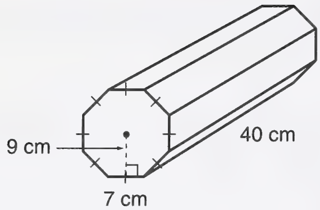
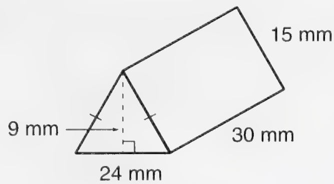
b.



6

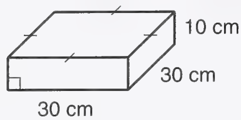
c.



2. Mrs. Smith baked a cake and iced the sides and top of the cake with frosting. The base of the cake is a square $30\ \text{cm} \times 30\ \text{cm}$; the cake is 10 cm high.

3 **a.** What is the volume of the cake?

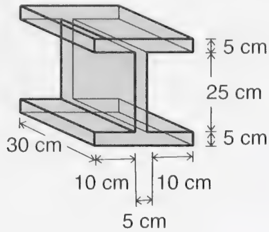
4 **b.** What is the surface area of the iced portion of the cake?



3 **c.** How can Mrs. Smith cut the cake into 8 pieces so that each person gets the same amount of cake and the same amount of frosting? Make a sketch.

5

3. A concrete block is shaped like this. What is its volume?



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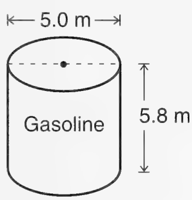
4. A painter paints the curved sides and flat top of this gasoline storage tank. If 1 L of paint covers 12 m^2, how much paint is required to cover the surface of the storage tank? **Hint:** Assume there is no wastage and you must purchase whole litres of paint.



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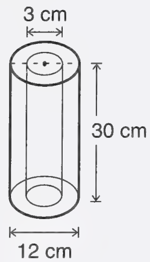
5. a. Paper towelling is wrapped around a hollow cardboard cylinder. In the given diagram, what is the volume of the paper towelling? **Note:** Round to the nearest tenth. (The diagram is not drawn to scale.)



2

b. Why do you think some manufacturers of paper towelling use a cardboard cylinder with a wider diameter than other towel rolls?

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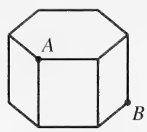
15

Final Module Assignment

Read all the parts of your assignment carefully and record your answers in the appropriate place. When answering the following questions, be sure to clearly show how you arrived at your answers.

4

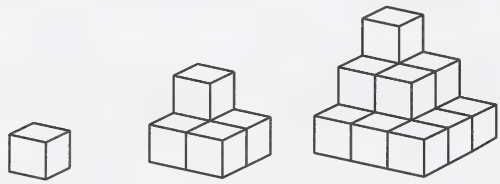
1. A spider wants to crawl from point *A* on the outside of an aquarium to point *B* on the outside. The aquarium is a hexagonal prism. Each side of the hexagonal base is 30 cm and the apothem is 25 cm. The height of the aquarium is 80 cm.



What is the length of the shortest path the spider can crawl to get from point *A* to point *B*?

2. Cubes that are $1\text{ cm} \times 1\text{ cm} \times 1\text{ cm}$ are stacked in square layers to form a pattern of towers as shown.

Tower 1 **Tower 2** **Tower 3**



1 cube 5 cubes 14 cubes

Hint: You may use sugar cubes, base ten blocks, or other small blocks to help you answer these questions.

Suppose you painted the exposed surfaces of each tower.

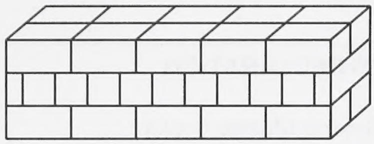
2 **a.** What is the area of the painted surfaces in the second tower?

2 **b.** What is the area of the painted surfaces in the third tower?

2 **c.** Draw a sketch of the fourth tower in the pattern.

2 **d.** What is the area of the painted surfaces in the fourth tower?

3 **3.** Bricks $8\text{ cm} \times 10\text{ cm} \times 20\text{ cm}$ are used to build this wall. What is the volume of the wall?



ASSIGNMENT BOOKLET DECLARATIONS

The Student's Declaration is to be filled in by a student registered at the Alberta Distance Learning Centre. If the student is under 16, the Learning Facilitator's Declaration is to be filled in by the learning facilitator. Failure to complete this page may invalidate the assignment results.

STUDENT'S DECLARATION

- I have followed the instructions outlined in the Student Module Booklet.
- I have completed the activities to prepare myself for the assignments in this Assignment Booklet.
- I completed the assignments in this Assignment Booklet by myself.

____

Student's Signature

LEARNING FACILITATOR'S DECLARATION

I hereby certify that I have supervised the learning activities completed by __________________________.

Student's Name

I also certify that to the best of my knowledge the assignments in this Assignment Booklet were completed independently by this student.

____

Supervisor's Signature

If you, the student or learning facilitator, have any comments or observations regarding this module, write them in the following space.

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